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Kant's
Philosophical Method (I)

Summary

The author claims that it is possible to look for the origins of Kant's philosophical method in the combined analysis and synthesis method of ancient Greek geometry. The combined method, consisting of analysis proper or transformation & resolution, constituting analysis, and construction & proof, constituting synthesis, enabled the Greek geometers to solve both theoretical and construction problems. The author states that Kant used the same method, adapted and transformed to a certain extent, during his critical period, as well as in order to solve the central problem of his transcendental philosophy, as presented in the first Critique and Prolegomena.

1. Introduction

Though it seems to be impossible to put all procedures of Kant's under one general methodological scheme, there are good grounds to say that most tenets of both his speculative and practical philosophy have been established by the method of analysis and synthesis adapted from Greek geometry. This way of looking upon Kant's royal road to philosophy sheds new light upon the structure of problems that he was solving as well as on the nature order and natural dependence of his arguments. Moreover, the understanding of Kant's method of analysis and synthesis helps greatly in the study of other methods employed by him.

**2. The Method
of Analysis and Synthesis**

The old combined method of analysis and synthesis of Greek geometers is briefly explained in an interpolation to Book XIII, Proposition 1, of the *Elements* of Euclid. Yet, the most complete description of it is found in Pappus's *Collectio*. I quote from this *locus classicus* of the ancient theory of scientific discovery in a recent translation given by Hintikka and Remes:

»Now analysis is the way from what is sought - as if it were admitted - through its concomitants in order to something admitted in synthesis. For in analysis we suppose that which is sought to be already done, and we enquire from what it results, and again what is the antecedent of the latter, until we on our backward way light upon something already known and being first in order. And we call such a method analysis, as being a solution backwards. In synthesis, on the other hand, we suppose that which was reached last in analysis to be already done, and arranging in their natural order as consequentes the former antecedents and linking them one with another, we in the end arrive at the construction of the thing sought. And this is synthesis. Now analysis is of two kinds. One seeks the truth, being called theoretical. The other serves to carry out what was desired to do, and this is called problematical. In the theoretical kind we suppose the thing sought as being and as being true, and then we pass

through its concomitants in order, as though they were true and existent by hypothesis, to something admitted; then, if that which is admitted be true, the thing sought is true too, and the proof will be the reverse of analysis. But if we come upon something false to admit, the thing sought will be false, too. In the problematical kind we suppose the desired thing to be known, and then we pass through its concomitants in order, as though they were true, up to something admitted. If the thing admitted is possible or can be done, that is, if it is what the mathematicians call given, the desired thing will also be possible. The proof will again be the reverse of analysis. But if we come upon something impossible to admit, the problem will also be impossible.«¹

This then is the combined method of analysis and synthesis of Greek geometers. As it is here described, it can be applied in solving two different kinds of problems, the theoretical and the construction problems. The unknowns of theoretical problems are *truth values* of conjectured theorems (or proof procedures for them) and those of construction problems are *objects* having certain properties (or construction procedures for generating them). Following Polya, I shall call the former *problems-to-prove*, and the latter *problems-to-find*.

The employment of the combined method in solving *problems-to-prove* can be reconstructed in the following way. There is a *starting move* in which we suppose the conjectured proposition to be true and the object which it is about to exist. The latter means that (in agreement with the standard practice of Greek geometers) we *instantiate* the proposition by constructing or, at least, by pointing to a case to which it applies.

This starting move constitutes the prelude to the *first half*, called *analysis*, of the whole combined method of analysis and synthesis. It has itself two parts. The first one, called *analysis proper* or *transformation*, consists of going upward in direction of that from which the conjectured proposition may result. Here we look for two kinds of antecedents: premisses (propositions) from which the proposition under examination may be deduced, and data from which the instance, by which the initial proposition was instantiated, may be constructed. We stop our movement upward when we come to legitimate premisses and data.

After this, the enquiry is continued by applying the second part of analysis, the so-called *resolution*. There we have to establish the legitimacy of the results reached at the end of the analysis proper, by proving the truth of the premisses and the givenness of the data at which we stopped.

The *second half* of whole method, called *synthesis*, consists again in two parts. The first of it is the *construction*, an operation by which we generate the figure which instantiates the conjecture proposition from data proved as legitimate (given) in the resolution. In the second part, named *proof*, we derive the initial proposition by starting with the premisses reached at the end of the transformation and legitimated in the resolution. Of course, various intermediary theorems may be needed for completing the proof.

This is in substance the standard interpretation of Pappus's description of the employment of the combined method in solving problems-to-prove, as it can be found, for instance, in Heath's Introduction to his edition of Euclid's *Elements*, and in Hintikka and Remes, *op. cit.* The following observations may be helpful for the understanding of its reception by Kant.

In the first place, premisses and data (constructions) hit upon in the analysis proper need not necessarily be evident first »principles«. Also

hypothetical principles can be considered. In that case, the solution of the problem will also be only hypothetical.

Secondly, there are different ways of going upward, as was pointed out by Hintikka and Remes. We can go upward by simply searching for premisses and data from which we may hope to deduce or construct, in the reverse order, the initial proposition. The other way of going upward is that of deducing acceptable premisses from the conjectured initial proposition and of actually constructing admissible data from the initial construction (instantiation). In order that the synthesis may work, we have to suppose the reversibility of the steps of the analysis. And if we light upon a false proposition or an impossible datum, we must conclude that the initial proposition or construction is also false or impossible, respectively. In this case, the method includes, therefore, the *reductio ad absurdum* technique.

Thirdly, since the transformation may involve both the operation of deduction and of construction, we can accordingly speak about propositional and constructional sense of analysis. Analogous remarks apply to the synthesis proper. Both senses are essential for the method as it was described by Pappus and practised by ancient and modern mathematicians.

Fourthly, constructions involved in the transformation are traditionally called auxiliary *constructions*. The heuristic fertility of the method is mainly due to them.²

The method of analysis and synthesis for *problems-to-find* has the same basic structure. In the first move we suppose the problem to be possible, i.e., solved, and instantiate its data, its unknowns and the condition which connects the former and the latter. The transformation consists of looking for presumably legitimate data from which the desired construction can follow. The second part of analysis is again the resolution which proves the legitimacy of the last data reached in the transformation. The synthesis starts only after the analysis has reached a satisfactory result. It is again twofold. Its first move is the construction of desired object and it ends after proving the legitimacy of the constructive steps executed. When constructions employed in transformation are reversible, the method yields the technique of *reductio ad absurdum*. In general, the reversibility of constructions makes the proof in the second part trivial.

In the present context the term »possible« has a specific and well defined meaning. It applies to any primitive object and any object that can be made, i.e., constructed by *postulates* or other construction procedures from primitive objects. Possible objects are also called »given« or »data«. The term »postulate« itself is also to be taken in the sense of Greek geometers. According to Geminus, a postulate is a demand to do (construct) something in a certain domain of objects which is easy to do (construct). In other words, a postulate prescribes »that we construct or provide some simple or easily grasped object for the exhibition of a character«.³ For example, »drawing a straight line from a point to a point is something our thought grasps as obvious and easy, for by following the uniform flow-

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Cf. Hintikka and Remes, 1974, pp. 8-10.

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Proclus, ed. Friedlein, p. 181.

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Cf. Hintikka and Remes, op. cit., pp. 32 and 38; cf. B 745.

ing of the point and by proceeding without derivation more to one side than another, it reaches the other point«. ⁴ We can follow the first postulate of Euclid »without any complicated process of thought«. ⁵ The same is true of other two Euclidean postulates which describe how to produce a finite straight line continuously in a straight line and how to describe a circle with any centre and radius, respectively.

There is an aspect of the combined mathematical method of analysis and synthesis for problems-to-find which is particularly important for the whole Kantian theory of knowledge, namely, that it can only be applied to data or objects upon which we can perform constructions. Since we can only perform such operations upon objects given in our intuition, the present method does not apply to objects given only in thought, in Kantian terms, to things in themselves. Since, on the other hand, mathematical propositions are endowed with necessity, mathematical constructions cannot be thought of as performed upon empirical objects either. I*~~46112~~ Therefore that in order to guarantee the constructibility of mathematical objects in the sense of Pappus one has to suppose that they belong to a domain which is at the same time phenomenal and non-empirical.

This consequence does not seem to have been drawn by Greek mathematicians themselves. But it was drawn by some philosophers. Proclus, in his famous commentary of Euclid's *Elements* concluded indeed that the domain of mathematical constructions must be the domain of objects that have existence in the imagination. ⁶ He also stated that figures, motions and properties of these objects must be recognized as being completely different from those of objects of the *nous* (things in themselves) as well as of objects of senses. ⁷

Proclus seems thus to have come very near the *theory of ideality* of space put forward later on by Kant according to which space is nothing other than an *a priori* condition of possibility of appearing. One of Kant's arguments starts with the question: *how do we arrive* at apodictic geometrical truths? In other words, *how do we come to know (erkennen)* necessary geometrical truths? (A 46-47). His answer is: only through construction of objects which are referred to by geometrical concepts. The next step of Kant's is to say that the required constructions cannot be empirical, for »no universally valid proposition could ever arise out of it« (A 48). It cannot be conceived as executed over things in themselves either, for if the geometrical object referred to in an apodictic proposition (for instance, a triangle) »were something in itself, apart from any relation to you, the subject, how could you say that what necessarily exists in you as subjective condition for the construction of a triangle must necessarily belong to the triangle itself?« (*ibid.*). In order to be knowable in an apodictic way, the constructible properties of mathematical objects must, accordingly, be subjectively necessary and objectively valid, which implies that they can only be valid for objects as they appear to us. What things-in-themselves may be we do not know, nor we need to know, for a thing can never come before us except in appearance (A277).

As we see, the present argument in favour of the ideality thesis of space, which itself is a central piece of the Kantian transcendental philosophy, is entirely based upon reflections concerning necessary conditions for finding and proving propositions in geometry and algebra by means of the

method of analysis and synthesis. This fact reveals the importance attributed by Kant to the combined method in mathematical praxis. It also stresses the very special significance of the same method for the understanding of the structure of the Kantian system. We shall try to establish an even stronger thesis, namely, that the combined method was considered by Kant as being the general method of discovery and proof in philosophy, as well as in mathematics.

3. Philosophical method in pre-critical writings

Already in pre-critical writings of Kant's the question of philosophical method is considered to be of crucial importance. The incompleteness and uncertainty in traditional metaphysics is attributed to the ignorance of adequate methods (Deut. A69; tr. p. 5; Nach. A B). The main source of error is said to consist of an unreflected imitation by philosophers of the axiomatic method of mathematicians. Bishop Warburton is right, Kant argues, in saying that nothing was more damaging to philosophy than the attempt to organize philosophical knowledge by starting with definitions and axioms and to prove all the remaining theses from these primitive elements (Deut. A 79; tr. p. 14). The main differences between philosophical and mathematical method concern precisely these primitive elements, as well as definitions and proof methods.

In mathematics, »elements« (concepts and propositions) are few and well established, while in metaphysics they are numerous and not well determined (Deut. *3). As to the definitions, in mathematics they are arbitrary conjunctions of primitive concepts and are produced before axioms and other propositions are formulated (*ibid.*). In philosophy, definitions are not arbitrary but based on analysis of concepts given *a priori*, mostly in a confused and undetermined way, in primitive propositions supposed to be already known (*ibid.*). Finally, mathematical proofs are based upon intuitive operations of synthesis over figures or symbols, whereas philosophical proofs result from discursive or formal operations over abstract concepts and propositions (Deut. *4). Because of all these differences philosophers would be well advised, Kant concludes, not to imitate mathematicians. They should rather try to follow the method of natural scientists, more precisely, the method commonly employed in Newtonian physics:

»The true method of metaphysics is basically the same as that introduced by Newton into natural science and which had such useful consequences in that field. It is said there that the rules, according to which certain natural phenomena occur, should be sought by means of certain experience and, if need be, with the help of geometry. Although the first principle is not perceived in the bodies, nevertheless it is certain that they operate according to this law. Involved natural occurrences are explained, when it is clearly shown how they are contained under these well proved rules« (Deut. A 82; tr. pp. 17-18).

4
Ibid., p. 179.

5
Ibid., p. 185.

6
Ibid., p. 51.

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Ibid., pp. 186-7.

The method recommended by Kant consists, as we see, of following three rules. Firstly, start with certain experiences. Secondly, search after »rules« (principles) which permit you to derive these experiences. Thirdly, employ these rules (principles) in constructing explanations for other, and more complicated, empirical events. The deduction of the principles from the essence or the nature of physical bodies is not a necessary condition of their employment in legitimate explanation. Such is the method which metaphysicians have to imitate:

»It is exactly the same in metaphysics: by means of certain inner experience, that is by means of an immediate evident consciousness, you ought to seek out those characteristics which certainly lie in the concept of any general condition; and, even though you do not know immediately the whole essence of the thing, yet you can still safely make use of it, in order to derive a great deal about the thing« (*ibid.*).

As we see, the parallel drawn by Kant between the best method in physics and in philosophy is perfect. Kant even gives a reason for this coincidence: the method in question reflects the way of the natural progress of human knowledge and is therefore the best method of teaching philosophy:

»The suitable method of teaching philosophy is *zetetic*, as it was called by some ancient thinkers, i.e. a method *which teaches how to make discoveries*, and it becomes *dogmatic*, i.e. *decided*, only when the reason is already well trained (...)« (Nach. A 6).

The present passage reveals that Kant knew quite well that the *zetetic*, that is, analytic method was already used in Greek antiquity. There is no doubt that he is alluding here to the first half of the well-known combined method of analysis and synthesis of Greek mathematicians. The Greek origin is implicitly admitted by Newton himself in his description of the method of natural science given in Query 23 of his *Opticks*:

»As in Mathematics, so in Natural Philosophy, the Investigation of difficult Things by the Method of Analysis, ought ever to precede the Method of Composition. The Analysis consists in making Experiments and Observations, and in drawing general Conclusions from them by Induction, and admitting of no Objections against the Conclusion, but such as are taken from Experiments, or other certain Truths... By this way of Analysis, we proceed from Compounds to Ingredients, and from Motions to the Forces producing them; and in general from Effects to their Causes, and from particular Causes to more general Causes, till the Argument ends in the most general. This is the Method of Analysis: And the Synthesis consists in assuming the Causes discovered, and established as principles, and by them explaining the Phaenomena proceeding from them, and proving the Explanation«.

The Kantian texts on the methods of natural science and of philosophy quoted above are faithful paraphrases of this well-known piece of Newton. According to Newton, the method enjoins us to take apart something given (a concept, an object, etc.), to go upwards in the direction of its conditions and to descend after that to the given or to other possible problem situations. In both descriptions the starting points and the operations involved are somewhat underdetermined (...) neither requires - and this is a very important epistemological point - that the synthesis be based on something certain or known. Newton is, however, more precise in at least one point. He distinguishes clearly between the method of analysis and the method of synthesis and alludes to the mathematical origin of both.

The *Dissertation* of 1770 provides us with some additional material concerning Kant's precritical views on the combined method. Kant's analysis and synthesis of the substantial *compositum* which is the world are said to have been based upon it (*1). Let me stress just one aspect of this

application of the method. The synthesis, that is, the genesis of the totality which is the world, is twofold in virtue of the double nature of our cognitive powers. It is produced both through the understanding and through the sensitivity. In the first case, the synthesis consists in a discursive operation over concepts and obeys the formal laws of the understanding, whereas in the second case it is executed by means of an intuitive operation over intuitions and obeys the conditions of time and of intuition in general (*ibid.*). This double-faced nature of the synthesis is a central aspect of this operation which has to be taken into account in all later contexts.

4. Kant's Description of Method of Analysis and Synthesis in the Critical Writings

In the critical period, the method of analysis is the actual basis of the great part of the Kantian methodological practice as well as of his theoretical considerations on the method in philosophy. The mathematical axiomatic method, on the other hand, continues to be rejected.

In the first Critique Kant explicitly denies that a philosophical doctrine can be axiomatized. The main differences between any reasonable method in philosophy and the axiomatic or dogmatic method of mathematics regard definitions, axioms and proofs. In philosophy, all primitive concepts are given before their definitions. The latter are accordingly always analytic and, therefore, insecure (B 760). In mathematics, no concept is given before its definition; all definitions are synthetic and can never be in error (B 759). Though primitive philosophical propositions can be completely enumerated just as primitive categories can, they are never immediately or intuitively certain. Mathematical axioms, on the other hand, are always intuitively indubitable, »since by means of the construction of concepts in the intuition of the object it can combine the predicates of the object both *a priori* and immediately« (B 760).

Finally, philosophical proofs are never strictly demonstrative, for their certainty is not intuitive, i.e., based upon intuitive axioms and intuitive proof-procedures. Only in mathematics there are demonstrations, »since it derives its knowledge not from concepts but from the construction of them, that is, from intuition, which can be given *a priori* in accordance with the concepts« (B 762). Moreover, from mere concepts not constructed in intuition it is not possible to prove any synthetic proposition. Since ideas of pure reason cannot be constructed at all, not a single synthetic proposition can be proved in its domain (B 761). The principles of the understanding are provable but, again, never »directly from concepts alone, but always only indirectly through the relation of these concepts to something altogether contingent, namely, possible experience« (B 765). The relation of concepts of the understanding either to actual or only to possible properties of experience does not make transcendental proofs of the principles of the understanding intuitive. They always consider the universal *in abstracto*, by means of concepts, and are always guided by the meaning of words alone or by the object of our thoughts. Therefore, they are better called discursive than intuitive (*ibid.*).

Kant's method of synthesis, suitable for scientific exposition and articulation of already found knowledge, is admittedly similar to the axiomatic method. It is also true that Kant has made some favorable remarks on the axiomatic method employed by Newton and has even tried to imitate it in his *Metaphysical Foundations of Natural Science* (1786).

Nevertheless, this imitation is only imperfect. In the whole of Kant's critical philosophy, the only general method continued to be the old method of analysis and synthesis.

One of the clearest descriptions of this method in the critical period is contained in the following text:

»Analytic method, in so far as it is opposed to the synthetic method, is something quite different from an aggregate of analytic propositions. It means that one starts from what is being looked for as if it were given, and ascends to the conditions under which alone it is possible. In this method one often uses nothing but synthetic propositions, as in the examples of mathematical analysis, and it might be better to call it the *regressive* method, in distinction from the synthetic or *progressive* method« (Prol. A 42 n; tr. p. 31 n).

There can be no doubt that the two methods distinguished and opposed by Kant are the first and the second half of the combined method of analysis and synthesis. In separating them, Kant follows a usage which can also be noted in the quotation of Newton's description of given above and which goes back to ancient mathematicians. Indeed, most Greek geometrical texts omit the analytic part of proofs and start immediately called progressive, or the method of composition or synthesis.

Kant's description of the two methods is rather vague about various issues. The given starting point is left unspecified, so that one may have doubts as to whether Kant is thinking of problems-to-prove or problems-to-find, and whether he takes the analysis in the propositional or in the construal sense. It is also not established whether the way upwards is reversible or not. Nothing is said about the nature of conditions aimed at in analysis. The opposition between the direction of regression and of progression is not made precise, nor is the synthetic way down.

Kant's scattered methodological remarks may help us to complete this admittedly partial picture of his philosophical method. The second edition of the first Critique brings a valuable note on the general applicability of both analysis and synthesis:

»In the systematic representation of ideas, the order cited (above), the synthetic, would be the most suitable; but in the investigation which must necessarily precede it the analytic, or reverse order, is better adapted to the purpose of completing our great project, as enabling us to start from what is immediately given to us in experience...« (B 395 n).

The project alluded to is that of solving the three basic unavoidable problems of pure reason, the question of necessary being, freedom and immortality (cf. B 7). This shows that for Kant the methods of analysis and synthesis are universally applicable in metaphysical problems. As to the difference between the two methods, it consists, according to the present remark, of the fact that analysis is a heuristic method or a method of growth of knowledge, whereas synthesis is a method of exposition of already found knowledge. Similar conclusion may be drawn from what is said about both methods in the *Logik Jaesche*:

»The analytic method, is also called the method of discovery (*Methode des Erfindens*). For the purpose of popularity the analytic method is more suitable; for the purpose of scientific, and systematic elaboration of cognition, however, the synthetic method« (LJ, * 117).

It would be a serious mistake, however, to conclude from these remarks that the analytic half of the combined method is of secondary importance and not scientific. For it is the property heuristic part of the combined method of discovery and proof, the execution of which is indispensable for initiating the synthetic or the proof part. Moreover, according to the *Prolegomena*, the analytic method indicates »what has to be done in order to bring a science into reality« (A 39; tr. p. 29). It provides in particular the plan and the guide for the transcendental research (Prol., a 218; tr. pp. 152-3). Before the end of the critical research this plan may well appear »unintelligible, unreliable and useless« to people who do not take part in transcendental research: but certainly not to Kant himself. And after the research is concluded and systematically exposed, it becomes much more useful to the reader without losing its usefulness for the transcendental philosopher in eventually improving the exposition:

»For it puts one in a position to survey the whole, to test one by one the main points that are important in this science, and to arrange some things better as regards the exposition than could happen in the first version of the work« (*Prol.*, A 20; tr.p. 13; cf. *Prol.*, A 218; tr. p. 153).

The analytic plan of his transcendental research is the main entry to the understanding of its general nature as well as of nature of the fundamental problem of the transcendental philosophy, as I shall show in detail below. It also plays a very important role in Kant's theory of pure reason. There is, indeed, a noteworthy parallelism between Kant's theory of employment of ideas in psychological and physical research (also called by Kant the method of ideas) and the method of analysis. Research guided by ideas consists in the construction of two sequences of propositions starting with a given empirical propositions. One of these sequences goes upward in the direction of empirical premisses from which the given propositions is logically deducible, and the other downward in the direction of logical consequences of the given propositions. No completeness is required for the second sequence. But for the first sequence pure reason foreshadows by means of ideas the complete, unconditioned, absolute totality of empirical premisses (conditions) of the initially given empirical propositions:

»The transcendental ideas thus serve only for ascending, in the sequence of conditions (premisses), to the unconditioned, that is to principles« (B 394).

This task is not a whim of our reason but is imposed by its very nature (B 384, 389). It is originally an innate *logical postulate* of pure reason (B 517), which imposes upon us the *logical* task »to find for conditioned whereby its unity is brought to completion« (B 384-5) The main natural objective of our cognitive apparatus is to solve this problem (B 393). However, this problem is demonstrably insoluble in the domain of empirical propositions. For there is no way to generate effectively the proposition (B 333). The only way to do this is to introduce non-empirical premisses and non-empirical unconditioned conditions, such as fundamental forces. These unconditioned conditions are precisely objects of ideas. They are not possible objects but rather necessary heuristic fictions to which we are conduced by logical constitution of our reason. That is in substance what Kant calls the subjective deduction of ideas (A XVII; B 393). Not referring

to possible objects, ideas are like *a priori* pigeon-holes for unknowns in problems of systematic unity of (propositions of) experience already characterized by the unity of the understanding. This systematic unity is to be achieved along the fundamental logical relations of predicate to subject, of ground to consequence and of disjunctive community (B 393) extended to the unconditioned concepts which are assumed to refer to the unconditioned (i.e., ideas), and to propositions containing these concepts (B 436).

Here we have enough conceptual evidence that the method of ideas is an adaptation of the method of analysis for the search after first principles is psychology and physics, and for the establishment of the one unified system of knowledge (B 394). there is also terminological evidence that this is so. As in the traditional descriptions of the combined method, in Kant's description of the method of ideas the upwards movement is called regress (B 469) and the downward movement progress (B 394, 438).

Even certain ambiguities are the same. So, for instance, here too the analysis may be of objective or of propositional data. The system of human reason as a system of ideas or idea-based principles is, therefore, a general framework for never-ending analysis of empirical knowledge aiming at producing its overall systematic unity. Kant's theory of innate logical organization of human reason is, in its most important aspects, a theory of the analytic method as an innate method of the human problem-solver.

I have just said that the passage from the *Prolegomena* quoted above leaves the question open whether the analytic and the synthetic method are meant in the propositional or in the constructional sense. That both meanings are admissible is explicitly said in Kant's remarks on this matter contained in the *Dissertation* which we have already commented upon. The same conclusion can be reached from the following passage in the *Logik Jaesche*, *117:

»The *analytic* method is opposed to the *synthetic* method. The former begins with the conditional and with what is provided and goes on to principles (*a principiatīs ad principia*); the latter goes from principles to consequences, or from the simple to the composite«.

Both senses are also allowed by the fact that the operations of synthesis employed are either intuitive or constructive, as the figurative synthesis is (B 151), or discursive, as is the case of synthesis in thought (B 365).

Foregoing considerations about the nature of the analytic method are the foundation of a basic element of Kant's theory of knowledge into knowledge from concepts and knowledge from the construction of concepts (B 741, 747, 203-4). This distinction is nothing more than a consequence of the difference between constructive and logical procedures involved in solving problems by methods of analysis and synthesis. Where does the construction of a concept consist in? »To construct a concept means to exhibit *a priori* the intuition which corresponds to the concept« (B 741). It is thus exactly the same operation as the one considered by Greek mathematicians, according to the interpretation of Kant himself, as explained above.

But for Greek mathematicians propositions too are constructible. The same is true in Kant (A 24, B 746). The construction instantiates them and these instantiations are employed in the discovery and formulation of

proofs and of objective unknowns. The following example of Kant's makes his views on this matter quite clear. Suppose that a mathematician is given the concept of triangle and is asked »to find out what relation the sum of its angles bears to a right angle« (B 744). As seen above, the combined method of analysis and synthesis tells him to begin by supposing the problem solved and by constructing a figure which exemplifies its data and the unknown relation. Kant's description of the mathematical solution procedure for this problem captures this and various other parts of the old method of analysis and synthesis. The geometer, says Kant,

»at once begins by constructing a triangle. Since he knows that the sum of two right angles is exactly equal to the sum of all the adjacent angles which can be constructed from a single point on a straight line, he prolongs one side of his triangle and obtains two adjacent angles, which together are equal to two right angles. He divides the external angle by drawing a line parallel to the opposite side of the triangle, and observes that he thus obtained an external adjacent angle which is equal to an internal angle - and so on. In this fashion, through a chain of interferences guided throughout by intuition, he arrives at a fully evident and universally valid solution of the problem« (B 744-5).

What is here described up to clause »and so on« is precisely the initial segment of the combined method which consists of instantiation and transformation. Kant implies that the transformation is both propositional (because it draws inferences) and constructional.

This last aspect of analysis is also recognized as essential for its heuristic fruitfulness. Kant himself compares the effectiveness of the traditional geometrical analysis with the philosophical. In the present case, what could a problem-solver achieve, supposing he were limited, like a philosopher, to propositional or discursive operations? He simply could not solve the problem, says Kant. He argues:

»He (the philosopher) has nothing but the concept of a figure enclosed by three straight lines, and possessing three angles. However long he meditates on this concept, he will never produce anything new. He can analyze and clarify the concept of a straight line or of an angle or of the number three, but he can never arrive at any properties not already contained in these concepts« (B 744).

In the present case (and of course in all similar cases) the discovery of new auxiliary constructions which exhibit new auxiliary properties of the triangle, which are not contained in its definition, are essential for the possibility of finding the solution. For this solution cannot be obtained by mere conceptual analysis of the concept given with the problem. It would be quite futile »to philosophize upon the triangle, that is, to think about it discursively. I should not be able to advance a single step beyond the mere definition, which was what I had to begin with« (B 746-7). In the present case I must, therefore, »not restrict my attention to what I am actually thinking in my concept of a triangle (this is nothing more the mere definition); I must pass beyond it to properties which are not contained in this concept, but yet belong to it« (B 746). By what means can I do that? It is impossible to do it legitimately in any other way than by determining the intuitive instantiation of the concept of the triangle »in accordance with the conditions of either empirical or pure intuition« (B 746). Mathematicians, of course, instantiate their definitions in pure intuition. Such determinations are exhibited on figures which can be generated in space and time through homogeneous synthesis in agreement with specific properties expressed conceptually. This process of synthesis or con-

struction is also characterized by certain universal formal conditions which are both subjectively necessary and objectively valid for any possible appearance (B 742, 744).

The comparison of Kant's proof theory and heuristics with properties of combined method of analysis is illuminating in many other respects. I shall point out only a few of them relevant to this study. The Kantian concept of *possible object*, being synonymous with the concept of object constitutable in empirical or pure intuition, is clearly a generalization of the concept of possible object as found in Pappus. Kant's apparently strange usage of term »possible« in question like »how are synthetic *a priori* propositions possible?« can here be traced back to its presumable origin. Just as objects are called possible if they can either arbitrarily constructed or exemplified in experience, propositions are called *possible* simply if they can be true (objectively valid) of possible objects. That is, if they are consistent propositions whose models can be constructed over the domain of sensibly possible objects. For Kant, as for the Greeks, the provability of a proposition is conditional on the possibility of instantiating it by an object which can be legitimately constituted. This is why Kant speaks not only of the construction of concepts but also of the construction of propositions. (It is therefore impossible to accept Erdmann's substitution of *erkannt werden können* for *construiert werden müssen* quoted on B 746, by Schmidt in his classical edition of the first Critique).

It is now very easy to characterize the difference between the methods employed in the *Critique of Pure Reason* and in the *Prolegomena*. The latter book describes the steps which come first from the methodological point of view, for it employs the method of analysis and shows how the elements of the Kantian theory of possibility of synthetic knowledge have been discovered. As Kant himself points out, the *Prolegomena* starts by considering »something that is already known as reliable«, namely, judgments of pure mathematics and physics, and confidently ascends »to the sources which are not yet known« (P, A 39). I take it for sure that, although published later, the *Prolegomena* describes the phase of Kant's transcendental research which chronologically preceded the results presented in the first *Critique*. This comes clearly out also from the historical remarks to be found in the introduction to it. Once he was in the possession of the source of synthetic knowledge found out in the analysis, Kant was ready to start writing the latter book by taking »as given nothing except reason itself« and by proceeding synthetically he could venture to »develop knowledge out of its original seeds without seeking support in any fact« (P, A 38). In doing so, Kant was able to explain the possibility of judgments which he already had the knowledge of and which he presupposed as real in the starting move, as well as to guarantee »a large extent of knowledge which springs exclusively from these same sources«, in particular, the transcendental and metaphysical knowledge (P, A 39). The methodological difference between the two books is thus the same as that which exists between the two parts of the combined method of analysis and synthesis. By imitating Greek geometers in that respect too, Kant published first his discoveries exposed synthetically, without disclosing the way in which he has lighted upon them or formulating the initial problem which has given rise to his entire research programme. Precisely this was done later on in

the *Prolegomena* and, to a much lesser extent, in the second edition of the first *Critique*.

5. A Kantian Problem

As an example of Kant's employment of the method of analysis in the critical period, I shall consider his treatment of the following problem: How are synthetic judgments *a priori* possible? This is, says Kant, the true question (*die eigentliche Aufgabe*) of pure reason (B 19) and therefore the main, nay, the only question of the whole transcendental philosophy (P, A 46). Since synthetic judgments *a priori* belong either to mathematics, pure physics or metaphysics, by giving a general answer to the question Kant also hopes to solve three related questions, namely, how pure mathematics, pure physics and pure metaphysics are possible as sciences (P, 47-8; B 202).

Kant has recognized that the present problem is a part of a still more general problem, namely to give an account of possibility of synthetic propositions *in general* (B 193). The latter is viewed already in the A edition of the first *Critique* as being the most important and actually the unique task of transcendental logic (B 193). Hence, transcendental philosophy must study the conditions of possibility of synthetic judgments *a posteriori* as well as those which are *a priori* (cf. Fort., A 49). It could not be otherwise, since the question about conditions of possibility is essentially related to the question of its logical form and logical forms of all synthetic judgments, be they *a priori* or *a posteriori*, are the same (p, A 121; tr. p. 87; cf. B 296).

This is the reason why Kant declares in the *Prolegomena* that the possibility of synthetic judgments *a posteriori* does not require a special explanation. Since these judgments originate from experience, that is, from the continuous synthesis of perceptions, their conditions of possibility are guaranteed by the same theory of the synthesis of perception which explains how *a priori* judgments are possible. In other words, this question brings no new difficulty to be considered. The fundamental problem of transcendental logic can therefore be solved in the restricted form in which it concerns only synthetic judgments *a priori*.⁸

What is it, then, that Kant means by »possibility« of a synthetic judgment? It seems to me that he means two different things. Firstly, the possibility of asserting (*behaupten*) it and, secondly, the possibility of coming to know it (*erkennen*), that is, of deciding it by proving or disproving it. On B 315 Kant explicitly treats the possibility of assenting a proposition as being a problem different from that of proving it. The distinction between mere asserting and deciding is also explicitly made on B 88 and 357. Moreover, according to B 223 the conditions of asserting are more fundamental and must be satisfied in order that a judgment might be proved.

⁸ In his transcendental studies Kant gives no special attention to analytic judgments either (cf. P, A 40). This is so because according to him the possibility of analytic judgments

is founded upon their formal agreement with the principle of non-contradiction. And in this respect Kant has nothing to add to the traditional wisdom in formal logic.

Now, what are the conditions of asserting a judgment? Clearly, no other than *truth conditions*. According to Kant, a judgment can be meaningfully asserted only if it is »possible«. And a judgment is possible only if it can be »objectively valid (real)« or »objectively invalid«. »Objective validity (reality)« and »invalidity« are in turn nothing other than the Kantian names for truth and falsity (B 269, 247, 194). Now to say that a judgment possesses objective validity (is true) is the same as to say that it *relates* to an object or that it has in this object its reference (*Sinn*) and meaning (*Bedeutung*). And a judgment of ours can relate in this sense to an object only if this object is *given* to us independently of the judgment itself. Since objects independent of particular judgments can only be given to us in sensible intuition, the only objects which we can meaningfully assert our judgments of are objects accessible to us in our sensible experience. In Kant, therefore, truth conditions of synthetic judgments are always understood as being conditions of their validity of empirical objects only and not as conditions of their validity of objects in general (cf. B 87-8).

The second kind of conditions of possibility of a synthetic judgment are, as it was said above, the conditions of coming to know whether the conditions of assertability are satisfied. They are conditions of proving or disproving, that is, of deciding it. From the Kantian treatment of the assertability conditions it follows that to prove a synthetic judgment is the same as to show, on the basis of possible intuition, that this judgment is true. This applies to mathematical and physical as well as to transcendental proofs (B 810-1). All of them are based upon the presupposition that truth conditions of propositions to be proved are always in principle accessible to our pure or empirical intuition.⁹

If this analysis is correct, one should expect Kant to have created a special discipline for the treatment of the question of truth conditions and decidability of judgments. In fact, he did, actually, this discipline is no other than his transcendental logic, whose declared task is to provide a logic of truth (B 87) as well as a canon of objectively valid employment of our understanding (B 170) and, secondly, the basis of a theory of proof for synthetic judgment in general (B 810-11).

The question about conditions of meaningful assertability and probability of synthetic propositions is important enough in itself. Historically, the discovery of its importance is due mainly to Kant. His interest in it has, on the one hand, been raised by Hume's demonstration of the improbability of the principle of causality by means of bothered about the question of provability and, indeed about a more troublesome one: very soon in his philosophical career (1769) he discovered his antinomies. Kant saw quite clearly from the very beginning (cf. the letter to Garve from 1798) that these paradoxes challenge our reason in a more radical way than Hume's criticism of traditional proofs of the principle of causality, since, of course, contradiction is a much worse threat again (...) of our capacity of producing reasoning than just the necessary ignorance of basic propositions of metaphysics.

6. Outline of Kant's solution

Let us go back to the problem of possibility of synthetic judgments in its restricted form which concerns only the *a priori* judgments. What is its solution in Kant? It can be summed up in the following way: There are certain intuitive and discursive representations (...) by *a priori* (or inborn) intuitive and discursive operations, which are *sufficient* for explaining the possibility of all objective knowledge. Intuitive representations in question are necessary *intuitive forms* or forms of all possible sensible (pure or empirical) intuitions, while discursive representations are *logical forms* of concepts and judgments, including concepts and judgments of theoretical philosophy, mathematics and science of nature (theoretical and empirical). The fundamental concepts, called categories, represent contents (objects) in so far as they are thought about in the forms of judgments. The problem of possibility of synthetic judgments *a priori* is thus reduced to that of the applicability of forms apply directly to the sensible manifold, since their construction (constitution) procedures, called schemata, are well defined for all sensible data; and, secondly, that *a priori* intuitive forms model logical forms of categories due to the fact that schemata themselves are defined by means of categories. Since categories represent contents which can be thought of in the forms of judgment, intuitive data (forms and contents) which satisfy categories can also be thought of or satisfy the forms of judgments which correspond to the categories. In that indirect way judgments are made applicable to the sensible manifold and thus objectively possible. Which means that they are provided with empirical truth conditions.

The harmony between intuitive and logical forms and, consequently, the possibility that sensible experience satisfies propositions having certain logical forms is established by a transcendental function or operation of the human mind. »The same function«, writes Kant, »which gives unity to the various representations in a *judgment* also gives unity to the mere synthesis of various representations in an *intuition*« (B 104-5). This function is allocated to the understanding:

»The same understanding, through the same operation by which in concepts, by means of analytical unity, it produced the *logical form* of a judgment, also introduces a transcendental content into its representations, by means of the synthetic unity of the manifold in intuition in general« (*ibid.*, my italics).

In that way all synthetic propositions, be they *a priori* or *a posteriori*, in which only occur objectively interpreted concepts and which have one of the logical forms producible by logic operations (which we shall call categorical), are ensured to be applicable in the domain of possible intuitions in general. In other words, they are ensured to be possible, i.e. to be objectively true or false.

9

Terms »objective validity« and »objective reality« are also used with concepts in order to say that they refer to objects that can be given to us (B XXVIn, 175, 288). Objectively valid concepts must therefore be distinguished from concepts which fail to refer to *phenomenal objects* though they may refer to

objects unspecified as to their mode of givenness (which are then called *objects in general*) or even to such objects which cannot possibly be given to us, as, for instance, *noumenal objects*.

This, then, is the solution of central problem of Kant's transcendental philosophy, as presented in the first *Critique* and in the *Prolegomena*. How was this solution found and how was it proved? The unquestionable answer is: by combined method of analysis and synthesis.

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Kants philosophische Methode (I)

Der Verfasser dieser Arbeit stellt die Behauptung auf, der Ursprung der philosophischen Methode Kants sei in der kombinierten analytischen und synthetischen Methode der antiken griechischen Geometrie zu suchen. Diese kombinierte Methode, bestehend aus einer eigentlichen Analyse bzw. Transformation und Auflösung (Resolution) und Synthese bzw. Auslegung und Beweis, erlaubte den Griechen, sowohl theoretische als auch Probleme aus dem Bereich der Konstruktion zu lösen. Der Verfasser stellt fest, daß Kant diese Methode in seiner kritischen Schaffensperiode benutzte, nur in abgewandelter und auf seine Bedürfnisse abgestimmter Form. Auch löste er mit Hilfe dieser Methode die zentralen Probleme seiner Transzendentalphilosophie, wie dies in der Kritik der reinen Vernunft und den Prolegomena dargelegt ist.

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La méthode philosophique de Kant (I)

L'auteur avance la thèse selon laquelle il est possible de découvrir les sources de la méthode philosophique de Kant en combinant les méthodes analytique et synthétique de la géométrie grecque. La méthode combinée comprend l'analyse - analyse première ou transformation et solution - et la synthèse - constructions et preuves et permet aux géomètres grecs de résoudre les problèmes théoriques et ceux de construction. L'auteur constate que Kant reprend la même méthode qu'il

applique transformée dans une certaine mesure, durant sa phase critique, en manifestant également l'intention de résoudre le problème central de sa philosophie transcendental, tel qu'il le présente dans sa première Critique et ses Prolégomènes.