The Logical Structure of the First Antinomy[1]

by Zeljko Loparic, Campinas/Brazil

I. *Introduction*

Like all other antinomies, Kant's first antinomy is a paradox of transcendental realism and is meant to provide a *reductio ad absurdum* of it. Yet, antinomies were not designed by Kant just to refute a doctrine which is the main rival of his transcendental idealism. For they belong to the critique of dialectical illusion and the central goal of that part of Kant's logical theory is to show that human reason "is burdened by questions which, as presented by the very nature of reason itself, it is not able to ignore, but which, as transcending all its powers, it is also not able to answer" (*Kr. d. r. V.* A VII)[2]. In other words, Kant's dialectic is essentially a critique of a priori methods of reasoning and proof. The specific critical contribution of the chapter on the antinomies to dialectic is that antinomic propositions violate the principle of the excluded middle with predicate negation. This result leads Kant to revise current logical and common sense views on predicate negation and on the employment of the method of indirect proofs as well as to reject the doctrine of transcendental realism.

II. *The Nature of the First Cosmological Problem*

It is therefore essential in studying antinomies to keep in mind the nature of the problems from which they arise. All of them are set up by a *logical postulate* and are therefore necessary and unavoidable. That postulate asks us to find for any particular item of knowledge (be it a concept or a proposition) the absolute totality of conditions from which it can be determined (B 364, 526). This fundamental principle of functioning of our reason can also be formulated as an *analytic proposition*. In this version the fundamental logical rule of our reason says that for any given conditioned item of conceptual or propositional knowledge the complete series of conditions, from which it can be determined (defined or derived), is also given (B 388, 444, 526).

---

[2] Henceforth the *First Critique* is cited with page-numbers only.

This same princple can still be given another propositional formulation, namely, the one which says that "if the conditioned is given, the whole series of conditions, subordinated to one another – a series which is therefore itself *unconditioned* – is likewise given, that is, *is contained in the object and its connection*" (B 364, my underlinings; cf. B 436). In this formulation, the fundamental principle of pure reason is clearly *synthetic*. It is synthetic, explains Kant, because a conditioned object "is analytically related to some condition but not to the unconditioned" (B 364). And it is also *transcendent* for the following reason. To say that such and such a conditioned object is given is to say that such and such an object exists. By applying the present version of the fundamental principle, we are therefore allowed to conclude from the existence of a conditioned object to the existence of the unconditioned, that is, to the existence of the absolute totality or whole of its conditions. The unconditioned inferred in that manner must accordingly be viewed as being given and as existing in the same sense or mode in which the conditioned object is taken to be given or exist. We see that the conclusion allowed by the present version of the fundamental principle "takes away" the limits of possible experience and, in addition, "commands us actually to transgress them" (B 353), which means that it is a transcendent proposition. The fundamental principle itself in the present version is therefore also a transcendent proposition. According to Kant, no objection can be raised against the employment of this formulation of the fundamental principle in speaking about things in themselves as given in pure thought, that is, either in non-schematised categories or in ideas of pure reason. It is only when it is applied to appearances that this formulation becomes one of the main sources of antinomies.

Let us now consider the particular problem which gives rise to the first antinomy. It is the first of *four* necessary cosmological problems of pure reason which ask us to find the series of all *antecedents* of any given *consequent*. There are just so many, neither more nor fewer, cosmological problems set to us by the fundamental principle of pure reason mentioned above, because, in view of the principles of the understanding, there are preisely four aspects of the a priori structure of an appearance which can be considered as *conditioned:* indeed, to the understanding, every appearance is, firstly, an extensive quantum, secondly, an intensive quantum, thirdly, an effect causally related to other such quanta and, fourthly, an accident as regards its existence.

The first cosmological problem is concerned with the first of these four aspects, namely, with appearances as *extensive quanta* (q_e). They are either temporal quanta (q_t) (events of finite series of events actually given in intuition or in successive synthesis of intuitions) or spatial quanta (q_s). It is required to find the unconditioned or absolute totality of their objective conditions. I shall call such totalities *Kantian worlds* (w), (DF$_1$). The absolute totality of objective conditions of a given temporal quantum consists itself of course of temporal quanta connected by the relation of succession. I shall call such temporal series *Kantian temporal worlds* (w_t), (DF$_2$). Analogously, the absolute totality of objective conditions of a given spatial quantum consists itself of spatial quanta which limit one another in order, while the first one limits the given spatial quantum. I shall call such series *Kantian spatial worlds* (w_s), (DF$_3$). Since

Kantian worlds of different extensive quanta contain different elements, the first cosmological problem splits in fact into an infinite number of sub-problems, which can be put in two classes: the class of temporal and the class of spacial sub-problems.

There are two important properties of Kantian worlds which we have to bear in mind when discussing the structure of the first antinomy. Firstly, these worlds are series which contain *only empirical members*, because only empirical antecedents can possibly be objective conditions of empirically given temporal quanta, and only empirical extensions can conceivably limit in an objective way finite empirically given spatial quanta. Kantian worlds which figure as data of sub-problems of the first cosmological problem are therefore always sensible or phenomenal and not intelligible worlds. Moreover, such worlds are all that is given to our reason in these problems (B 461). We cannot therefore abstract from the condition of sensibility without destroying the whole first cosmological problem itself.

Secondly, the *connection* between members of Kantian worlds must also be given in possible experience. In the present case the only possible experience left is the successive synthesis of intuitions, because no temporal or spatial world can ever be given in just one intuition (B 458). The successive synthesis appropriate here is that of *empirical regress* (B 541), an operation which ascends in serial order from something given as conditioned to its conditions, as opposed to empirical progress which descends from the condition to the conditioned (B 539–40).

This, then, is the basic structure of the cosmological problem about the magnitude of the world. Let us now see how it gives rise to the first antinomy of pure reason.

III. *The Structure of the First Antinomy*

The temporal and spatial antinomies arise from an *experiment (Fortschritte* A 95/ Ak XX, 291)[3] made by Kant to solve temporal and spatial cosmological sub-problems by a priori means of pure reason not submitted to a previous criticism of their problem-solving power. These means are formal logic and the basic tenets of transcendental realism.

Formal logic enters the experiment with six principles, namely

L_1 $\neg (Pa \wedge \neg Pa)$
L_2 $Pa \vee \neg Pa$
L_3 $\neg (Pa \wedge P'a)$
L_4 $Pa \vee P'a$
L_5 $((P \rightarrow Q) \wedge P) \rightarrow Q$
L_6 $((P \rightarrow Q) \wedge \neg Q) \rightarrow \neg P$

---

[3] *Welches sind die wirklichen Fortschritte, die die Metaphysik seit Leibnizens und Wolffs Zeiten in Deutschland gemacht hat?* (1804), cited with the original pagination, followed by the vol.-and-page-numbers of the academy edition. Likewise the *Prolegomena* and the *Logik Jäsche,* if not cited with §-numbers.

L_1 und L_2 formalise the two parts of what Kant calls the principle of excluded middle *(Logik Jäsche* § 48). This principle says that two propositions one of which is the propositional negation of the other, cannot be both true (L_1) nor both false (L_2). In keeping with the modern usage, I shall call L_1 the principle of non-contradiction and L_2 the principle of excluded middle, with propositional negation. L_3 and L_4 symbolise these same principles with predicate negation. Finally, L_5 and L_6 stand for the *modus ponens* and the *modus tollens,* respectively. These two rules are parts of the Kantian logical principle of sufficient reason (*Logik Jäsche* A 73–5 / Ak IX, 51–53). Notice that L_2 is equivalent to "$\neg\, Pa \rightarrow \neg\, Pa$" (principle of identiy) and to "$\neg\, \neg\, Pa \rightarrow Pa$" (principle of double negation). L_4 in turn is trivially equivalent to "$\neg\, Pa \rightarrow P'a$" and "$\neg\, P'a \rightarrow Pa$".

Although these principles apply to propositions of any form, I have found it more expedient, in the present context, to restrict them to predicative propositions and their compounds only. In all formulas, the symbol "a" stands for different kinds of subject terms such as proper namens ("Socrates"), demonstrative-plus-noun-phrases ("this man") and definite descriptions ("the sensible world", "the sum of all appearances", "the absolute totality of conditions of a given conditioned"). I want to stress that (for reasons which will become clear below) the existential analysis of definite descriptions *à la* Russell does not provide an adequate account of the Kantian employment of these expressions. In Kant, sentences like "The sensible world is finite" are treated as predicative propositions which implies that Kant considers definite descriptions as being genuinely referring expressions. Notice also that the Kantian subject terms are allowed to refer to members of the *domain of phenomenal entities* (D_p) and to the *domain of things in themselves* (D_t), as well as to classes and series of members of both of these domains. As we shall show below, members of D_p are specified in terms of empirically significant concepts and those of D_t through non-schematised concepts of the understanding or of reason.

Transcendental realism contributes with the following tenets:

R₁   T-thesis: Appearances or objects which are or can be given to us in possible experience are self-subsistent entities (B 519)

R₂   I-thesis: Self-subsistent entities can be given in pure intellect

The point of the T-thesis is not just that empirical objects exist, but that "appearances, and the sensible world which comprehends them all, are things in themselves" (B 536). It is not a thesis about existence of empirical things but about their *mode of existence.* Let us call this mode "T-mode". Kant's transcendental idealism, which denies the T-mode of empirical objects and of the sensible world, claims instead that all objects of any experience possible to us are "nothing but appearances" and thus "mere representations", which have "no independent existence outside our thoughts" (B 518–9). We can therefore say that transcendental idealism views all empirical things and their compunds in the phenomenal key or mode ("P-mode"). I stress that to deny of appearances and the sensible world that they exist in the T-mode is not the same as to deny that they exist in the space outside ourselves. Kant actually provides an elaborate proof of existence of external objects (B 274 ff.).

The basic difference between entities characterised by the T-mode and those characterised by the P-mode of existence consists in their relation to time and space: whilst T-objects are thought to exist in objective time and space (cf. B 563), P-objects are said to exist only in temporal and spatial relations which are subjective conditions of our sensible intuition.

The point of the I-thesis is that non-schematised a priori concepts, such as categories of the understanding and ideas of reason are *objectively significant* in the domain D_t and that propositions in which such concepts occur are *objectively valid,* that is to say, have definite truth values. In other words, the I-thesis implies that members of D_t are no other than noumenal entities in the Kantian sense and that propositional knowledge of such entities is possible. The I-thesis can therefore said to be about the *mode of givenness* of self-subsisting objects *in our intellect.*

In opposition to the I-thesis, transcendental idealism holds that there is no possible propositional knowledge about members of D_t, which does not prohibit us from thinking about them in non-schematised concepts under the guidance of the principle of non-contradiction. Nor can we do otherwise. For members of D_t are *entia rationis* given through logically possible but void concepts (B 348, 625n, 704–5).

The main datum which the first cosmological problem is about, the sensible world, is given by means of the transcendent version of the fundamental principle of pure reason. It can of course also be attributed to transcendental realism. In the present case, this principle can be formulated as follows:

R₃   If an extensive, temporal or spacial, magnitude ($q_{t,s}$) is given, the whole or complete series of its objective conditions subordinated to one another, a series which is therefore itself unconditioned ($w_{t,s}$) – is likewise given, that is, is contained in the object and its connection.

All of these are a priori ingredients of the first cosmological problem. The only a posteriori ingredients are the extensive temporal and spatial magnitudes ($q_{t,s}$), *given* in experience in an indefinite number. They are the data from which the main datum of the first cosmological problem, the sensible world, is obtained.

Let us now reconstruct the steps by which the first cosmological problem can be formulated fom these elements. The first step must consist of course in recognizing the givenness of $q_{t,s}$, that is, the existence of empirical objects in time and space. The second step applies the T-thesis to these data. From now on, we shall be speaking about self-subsisting things, that is, things existing in themselves or in objective time and space. The third step consists in applying the transcendent version of fundamental postulate of pure reason to these data. The immediate conclusion is that indefinitely many *complete* series of their objective conditions, that is, Kantian temporal and spatial worlds ($w_{t,s}$) are also given and contained in them and in their connections. Which, in turn, implies that the $w_{t,s}$ as well the $q_{t,s}$ belong to the domain D_t of things in themselves. As a consequence, the $w_{t,s}$ exist in objective time and space, and are therefore actual or completed absolute totalities and not totalities in becoming. Such totalities or wholes are the main data of the first cosmological problem.

We must remind us here that according to Kant, no objection can be raised against the employment of the fundamental principle of pure reason as regards things in themselves. If one accepts R_1, as a transcendental realist does, the conclusion arrived at in the third step is also entirely unobjectionable.

The fourth step consists in noticing that since each $w_{t,s}$ is a thing in itself, that is, belongs to the domain D_t of self-subsisting entities, it can, by the I-thesis, be given in pure thought and in particular, in non-schematised category of quantity.

In the fifth place, since the predicate "finite" (F) is obviously meaningful in D_t, we can conclude, by employing L_2 and L_4, that the following formulas

R_4 $F_{wt,s} \lor \neg F_{wt,s}$, and

R_5 $Fw_{t,s} \lor F'w_{t,s}$

express valid propositions. Notice that in the present context "F" and "F'" stand for "actually finite" and "actually infinite". This is a direct consequence of the supposition that we are talking about magnitudes contained in D_t. The exact phrasing of the thesis of the temporal antinomy, for instance, is now: "The sensible world existing in itself is actually finite." All of the antinomic propositions must also be understood in this more precise sense.

R_5 thus follows directly from R_1, R_2 and L_4. Notice that it can also be obtained from R_1, R_2 and L_2 by means of definitions of finite and infinite magnitudes in D_t. According to the traditional formal logic, any concept can logically be divided by means of pairs of concepts which obey the following conditions: firstly, they are contradictory opposites, which means that they can be generated from one another by pure logical predicate negation and that they satisfy the law of excluded middle (and of non-contradiction); secondly, that they both belong under the divided concept in the sense that they apply to one part or another of its sphere; and, thirdly, that their spheres taken together are equal to the sphere of the concept to be derived (*Logik Jäsche* §§ 110, 111).

Let us now suppose that the concept of extensive magnitude is given a realist interpretation. Then, F and F' logically divide it, for F and F' are contradictory opposites. Firstly, on account of the I-thesis, the operation of formation of the complement of a predicate (') must be taken here as an operation of pure thought, that is, as the classical formal predicate negation, which obeys classical laws of non-contradiction and excluded middle. Secondly, both F and F' obviously apply to magnitudes which exist in themselves and are given in pure concepts. Thirdly, the union of the spheres of F and F' equals the sphere of self-subsisting magnitudes. For, by supposition, the latter sphere contains extensive magnitudes given as complete in pure thought without any limitation through subjective time or any concept of succession (cf. B 528). Now, a complete extensive magnitude is defined as being constituted of actual self-subsisting homogenous and equal parts or units. If such magnitude contains a quantity of parts which is less than or equal to a given number, then it is called (actually) finite (DF₄). If it contains a quantity of parts which is not less nor equal to any given number, then it is called (actually) non-finite or infinite (DF₅). Since a

number can be given by successive synthesis, an actually infinite extensive magnitude cannot, by DF_5, be given by this operation.[4]

In virtue of L_2, it is analytic that a given magnitude either contains or does not contain a quantity of parts equal to or less than a number, it is also analytic that it is either finite or infinite. The following is therefore an analytic formula in the domain of self-subsisting extensive magnitudes (q_c):

R_6 $Fq_c \vee F'q_c$

Since, by virtue of the T-thesis, Kantian worlds are self-subsisting extensive magnitudes, the same formula must be valid in their case, which establishes R_5. By combining R_4 and R_5, we obtain

R_7 $Fw_{t, s} \vee (\neg Fw_{t, s} \wedge F'w_{t, s})$

as being an analytic disjunction.

It is easily seen that R_7 formalises the *Kantian statement* of both the temporal and the spacial antinomy. For the former consists in saying that the empirical world either has a beginning in time (is finite as regards time) or it has no beginning, *but* is infinite (non-finite) as regards time, while in the latter we have to choose between the thesis that the empirical world is limited as regards space, and the antithesis which claims that it has no limits in space, *but* is spacially infinite (non-finite) (B 454). We see that in Kant's formulation of the first antinomy the antithesis is a conjunction of a propositional negation of the thesis and of a limitative proposition obtained from the thesis by negating its predicate. I shall accordingly speak of the *negative* and *limitative* (forms of the) antithesis.

Since R_7 is, under realist assumptions, an analytic disjunction, it is possible to prove any side of it by employing the indirect method, that is, by falsifying one side and concluding by L_4 and L_6 that the other one is true. Since the first cosmological problem is not an empirical problem and thus cannot be solved by considering actually given properties of Kantian worlds, the only way of falsifying any side of the above disjunction is to show that it leads to a contradiction.

This is, I submit, Kant's *design of the experiment* which leads to the first antinomy. Kant claims (B 535; *Prolegomena* §§ 52, 52 b), that both the formulation of the first cosmological problem as well as the method of solution are entirely *unobjectionable* to anyone who accepts the basic tenets of transcendental realism and the classical formal logic. If the experiment were successful, that is, if the first cosmological problem could be solved by these tools it would be a case for believing that our pure reason is capable of acquiring new synthetic knowledge about nature in itself by entirely a priori means. Yet, the experiment fails, because it leads to the violation of the law of non-

---

[4] The first antinomy has nothing to do with paradoxes of mathematical infinite. This concept of infinite, which is identical to the realist concept of complete or actual infinite, coexists peacefully with Kant's own transcendental concept of infinite as a quantum the units of which can never be completed through the successive synthesis (B 460). Cf. also Kant's defense of the concept of actual infinite in *Dissertation* § 1 n.

contradiction (L_1 and L_3). The set L_1 to L_4 and R_1 to R_3 of principles is thus not self-consistent. There is only one conclusion left, namely, that the transcendental realism and the traditional formal logic do not constitute an a priori organon for successfully solving necessary problems about quantitative a priori aspects of the sensible world.

IV. *Kant's proof of the temporal antinomy*

Kant's claim that his proofs of the fourfold antinomy of pure reason are well-grounded if we accept the framework described above (B 535), has often been challenged, in particular, by British commentators. It seems to me that most of these objections are due to the lack of clarity as to the logical structure of Kant's arguments and justification for each of the steps. In order to settle the issue, I shall offer in the following a precise analysis of Kant's proofs of the temporal antinomy. Proofs of other antinomies can be reconstructed along similar lines.

Proof 1: proof of the temporal thesis (B 454):

(1) The world has no beginning in time ($\neg \, Fw_t$)
(2) Up to every given moment an eternity has elapsed (Ew_t)
(3) There has passed away in the world an infinite series of successive states of things ($F'w_t$)
(4) The infinity of the series consists in the fact that it can never be completed through successive synthesis
(5) It is impossible for an infinite world-series to have passed away ($\neg \, F'w_t$)
(6) A beginning of the world is a necessary condition of the world's existence (Fw_t).

Justification of the steps:

(1) Assumption of the propositional negation of the thesis, by indirect proof method and L_2.
(2) From (1) by R_5 and by taking eternity (E) for temporal infinity
(3) From (2) by DF_5
(4) DF_5
(5) DF_5 and DF_2[5]
(6) From (5) by R_5.

In this proof it is the limitative form of the antithesis (F'wt) which is reduced to absurdity in the first place. Its propositional opposite ($\neg \, F'w_t$) is reached in three moves (steps 3 to 5) just by applying the definitions of infinite series and of Kantian temporal world (DF_5 and DF_5). It is thus proved to be self-contradictory for semantical reasons. These reasons reduce in substance to the claim that a series of *sensible* elements cannot possibly be *actually* infinite. On the other hand, the propositional opposite (Fw_t) of the negative form of the antithesis ($\neg \, Fw_t$) which is reached only in the last

---

[5] Cf. Kant's comments on this aspect of the proof: "The true transcendental concept of infinitude is this, that the *successive synthesis* of units required for the enumeration of a quantum can never be completed. Hence it follows with complete certainty that an eternity of actual successive states leading up to a given (the present) moment cannot have *elapsed*, and that world must therefore have a beginning" (B 461, my italics). Kant's point is simply that first cosmological w_t which is, as we said above, the only datum of the problem, cannot possibly be considered as actually infinite because it is an empirical series.

step (6) is not justified by definitions, but by R_5. Its absurdity does not follow therefore from the meaning of concepts which it contains, but from the presupposition that R_5 is a valid rule of reasoning about the magnitude of w_t.

Proof 2: proof of the temporal antithesis (B 454):

(1) The world has an beginning in time (Fw_t)
(2) There must have been a preceding time in which the world was not, that is, an empty time.
(3) No part of such a time possesses, as compared with any other, a distinguishing condition of existence rather than of non-existence; and this applies whether the thing is supposed to arise of itself or through some other cause.
(4) No coming to be of a thing is possible in an empty time.
(5) The world has no beginning in time ($\neg\ Fw_t$)
(6) The world is infinite in respect of past time ($F'w_t$)

Justification of the steps:

(1) Assumption of the thesis, by the indirect proof method
(2) From (1) by explanation of the concept of beginning or of limit of a temporal world
(3) From (2) by definition of empty time and of mode of existence of w_t in objective time and space.[6]
(4) From (3) by same reasons
(5) From (4) by L_2
(6) From (5), by R_5.

This proof starts with the thesis (Fw_t) in order to conclude on line (4) to its falsity for reasons which have to do with the mode of existence of the datum of the problem under discussion. Its propositional negation, that is, the negative form of the antithesis ($\neg\ Fw_t$), is established immediately afterwards (line 5), by applying L_2. The limitative form of the antithesis ($F'w_t$) is reached last, by means of R_5. If we compare the two proofs with one another, we see that, taken together, they provide a *reductio ad absurdum* of both sides of R_5 and R_7 and, therefore, also of these propositions themselves. Which is a bad surprise for the transcendental realist who must have expeced that at most one of them would come out as being logically untenable. The attempt at solving the first cosmological problem by indirect proof method within the framework of transcendental realism ends thus in disaster: not only is the problem left without solution but it has been proved that the theoretical framework of transcendental realism is self-contradictory.

V. *Comments on some commentators*

Commentators I know of have failed to pay attention to the fact that the antithesis of the first antinomy is a conjunction of the propositional negation of the thesis and a

---

[6] Kant comments: "But we are here treating only of the *mundus phaenomenon* and its magnitude, and cannot therefore abstract from the aforesaid conditions of sensibility *without destroying the very being of that world.* If the sensible world is limited, it must necessarily lie in the infinite void. If that void, and consequently space in general as *a priori* condition of the possibility of appearances, be set aside, *the entire sensible world vanishes.* This world is all that is given us in our problem" (B 461, my italics).

limitative proposition obtained from the thesis by substituting for the predicate ("finite") its contradictory opposite ("infinite"). Commonly, they just render, like Russell[7] does, the Kantian text without any further thought on the logical form of propositions which enter the antinomy. Others, like Kemp Smith[8], Broad[9] and Gram[10] satisfy themselves by what seems to be a mere paraphrasis of Kant's statement. It is even more puzzling that a logically sophisticated commentator such as Strawson[11] procedes as though he could spare himself the effort of making a clear statement of the first antinomy. In his discussion of Kant's proofs the temporal antithesis is treated as a negative proposition (Strawson, *The Bounds of Sense*, p. 77) and the spatial as antithesis as a limitative proposition (*ibid.*, p. 182). The difference is left unnoticed. Again, Al-Azm[12] maintains that the antithesis of the first antinomy is the claim to the effect that the world is an infinite given whole of existing things. This claim, however, is not a part of the antithesis itself but of a line of an indirect proof of it. Even authors like Llewelyn[13] and Walsh[14], who have applied themselves to show what dialectical opposition between the antithetic propositions of the antinomies consists in, failed to make any connection between the dialectical opposition and the quality of these propositions.

Allison, in his otherwise excellent recent book on Kant's transcendental idealism,[15] affirms that the antithesis of the first antinomy denies that the world has either a beginning in time or a limit in space and contends instead that the world is infinite with respect to both time and space. Why "instead"? As we have seen, the antithesis *both* denies the thesis and asserts the actual infinity of the sensible world. This simplification of the problem of the first antinomy leads Allison to the conclusion that the dispute underlying the first antinomy can be symbolised as "($\exists$ x) (Fx v Ix)", where "F" stands for finite and "I" for infinite with respect to both time and space. Undoubtedly, this is an oversimplification of the first cosmological problem. Allison himself seems to feel obliged to defend his proposal and advances in a note the following justification for it: "I have symbolised the antithesis position as I (for infinite) rather than as $\neg$ F (for not finite) because the antithesis argues for actual infinite, rather than claiming merely that the world cannot be considered as finite in relevant aspects. In fact, we shall see that this

[7] B. Russell, *The Principles of Mathematics*. London: G. Allen and Unwin, 1979 (first published in 1903).

[8] N. Kemp Smith, *A Commentary to Kant's Critique of Pure Reason*. New York: Humanities Press, 1923.

[9] C. D. Broad, *Kant's Mathematical Antinomies*, Proceedings of the Aristotelian Society, 1954–5.

[10] M. S. Gram, *The Transcendental Turn*. University Press of Florida, 1984.

[11] P. F. Strawson, *The Bounds of Sense*. London: Methuen, 1966.

[12] S. J. Al-Azm, *The Origins of Kant's Arguments in the Antinomies*. London 1972.

[13] J. E. Llewelyn, *Dialectical and Analytical Opposites*, Kant-Studien 55 (1964), 171–4.

[14] W. H. Walsh, *The Structure of Kant's Antinomies*, in Pierre Laberge *et al.* (Eds.): *Proceedings of the Ottawa Congress on Kant in the Anglo-American and Continental Traditions, October 10–14, 1974*. Ottawa: The University of Ottawa Press, 1976.

[15] H. E. Allison, *Kant's Transcendental Idealism. An Interpretation and Defense*. Yale University Press, 1983.

is precisely what is wrong with the position" (Allison, *Kant's Transcendental Idealism*, p. 337). But this is scarcely an acceptable justification for the symbolism proposed. Firstly, the first cosmological problem is a necessary problem of pure reason, constituted by purely a priori means, about empirically given temporal and spatial quanta. Since there are a priori two alternatives of the thesis that the sensible world is actually finite, namely the negative and the limitative form of the antithesis, both must be considered in the statement of the problem, except of course if they are proved to be equivalent. Yet, no such proof is attempted by Kant, for the good reason that they are not equivalent, as we shall show presently. Secondly, the correct statement of the dispute in which the realist is involved can only be a disjunction of predicative statements and not a single existential statement, for otherwise the strategy of indirect proof cannot be applied at all. Thirdly, the elimination of the antithesis in its negative form cannot be justified just by saying that it is unobjectionable. Finally, this unobjectionable negative form of the antithesis actually appears in several steps in both proofs of the antinomy.

The lack of clarity as to the very formulation of the problem underlying the first antinomy when added to a deficient reconstruction of the transcendental realist framework within which Kant's proofs are produced, is responsible for most of recent criticism of Kant's derivation of the first antinomy. Early Russell[16], for instance, is of the opinion that the thesis of the first antinomy "is not concerned with pure time and pure space but with the things in them" (Russell 1979, p. 459). He is wrong in so far as the thesis is about the sensible and not the intelligible world. Again, Kemp Smith seems to imply that the problem of the first antinomy is the size of pure time and space and not of the sensible world. The intermediate Russell believes that Kant professes to prove *(sic!)* both the thesis and the antithesis of the first antinomy, whereas, he adds, in the light of modern logic, it must be impossible to prove any of them (Russell, *Our Knowledge*, p. 160). Even the old logic of the time of Kant is sufficient to show, that no such proof is possible: the first antinomy is, as we have seen, an attempt at *disproving* both. In the same text Russell adavances an objection against the step (5) of the Proof I, which follows, as we have seen, from the definition of the Kantian temporal world and the definition of the infinite. Russell argues that "when Kant says that an infinite series can 'never' be completed by successive steps, all that he has even conceivably a right to say is that it cannot be completed *in a finite time*" (*ibid.*, 161). Therefore, he continues, "what he really proves is, at most, that if the world had no beginning, it must have already existed for an infinite time" (ibid.). Which is a poor objection because it simply ignores the very datum of the first cosmological problem: it is *mundus phaenomenon*, that is, the series $w_{t,s}$. As we have seen, such series must be constituted in a possible experience. From where it follows that they cannot possibly have existed for an infinite time.

---

[16] B. Russell, *Our Knowledge of the External World*. London: Routledge and Kegan Paul, 1957 (first published in 1914).

Russell even ventures an explanation of this alleged Kantian error in his usual commonsensical vein: "Owing to the inveterate subjectivism of his mental habits, he [Kant] failed to notice that he had reversed the sense of the series by substituting backward synthesis for forward happening, and thus he supposed that it was necessary to identify the mental series, which has no end, with physical series, which has an end but no beginning" (ibid.). Here again Russell misses the point. Kant is not confusing the mental series with the physical one, but arguing that it is self-contradictory to say that a series given in possible experience, such as a temporal or spatial sensible world, possesses properties, which, like the actual infinity, cannot possibly be given in any experience.

Strawson, applauded by Bennett[17] and by others, echoes Russell's objection by producing the following variation of it: "A temporal process both completed and infinite in duration appears to be impossible only on the assumption that it has a beginning. If [...] it is urged that we cannot conceive of a process of surveying which does not have a beginning, then we must inquire with what relevance and by what right the notion of surveying is introduced into the discussion at all" (Strawson, *The Bounds of Sense*, p. 177). Here again the shot goes astray. Kant is not considering the question of conceivability of temporal processes in general but whether an empirical series constitutable in empirical regress performed in agreement with the fundamental postulate of pure speculative reason can be completed and thus actually infinite. His point is that it cannot.

Strawson does not appear to be aware of the logical structure of the antinomies either. According to him, what is established by the arguments of the antinomies taken together is that (1) if the world series existed as a whole, it would exist as a limited whole, and (2) if the world series existed as a whole, it would exist as an infinite whole *(ibid.,* p. 187).

As we have seen, none of these propositions is *proved* in the first antinomy. Proofs offered by Kant *presuppose* the assumption that if the sensible world exists as a thing in itself, it would be either actually finite or actually infinite in extension as regards time and space. The presupposition is different from the conjunction of (1) and (2). What, on the other hand, Kant's proofs do establish is that the sensible world (existing in itself) is neither actually finite nor actually infinite, which is of course again something different from (1) and (2).

VI. *Kant's criticism of L₄*

The first antinomy shows that the set of principles L_1 to L_4 and R_1 to R_3 is inconsistent. Is it possible to determine which of these principles must be rejected? Let us consider logical principles first. From Kant's proofs above it can easily be shown that

---

[17] J. Bennett, *Kant's Dialectic*. Cambridge: Cambridge University Press, 1974.

the first antinomy offers a counter-example to L_4. By abandoning it we preserve the validity of all other laws.

According to Proof II, the thesis (Fw_t) ist self-contradictory and, according to Proof I, the limitative form of the antithesis ($F'w_t$) has the same defect, and for the very same reason: both are inconsistent on ground of definitions of concepts which they contain. Their disjunction (R_5) is therefore necessarily also false and is thus a case against L_4).

The negative form of the antithesis ($\neg Fw_t$), on the other hand, is not proved to be self-contradictory on ground of definitions, neither in Proof I nor in Proof II. The line (5) of Proof II establishes, to the contrary, that it is necessarily true, being the propositional negation of a semantically self-contradictory proposition, namely, of the thesis. As to the propositional opposite of this form of the antithesis reached on line (6) of Proof I, it is established by employing R_5, a move which, in the light of the antecendent steps must be considered as illegitimate. If R_5 is rejected, no counter example for L_2 can be generated and the formulas

R_8 $Fw_t \vee \neg Fw_t$
R_9 $F'w_t \vee \neg F'w_t$

continue to be valid[18]. Clearly the same move preserves the validity of L_1 and L_3.

That L_4 is not universally valid can be followed already from Kant's doctrine on the quality of judgments, which at its face value is quite independent form the first antinomy. According to this doctrine, the propositional negation ($\neg Pa$) of a predicate proposition (Pa) is not equivalent to the limitative proposition ($P'a$) obtained by negating its predicate (P). That is to say, the formula

L_7 $\neg Pa \leftrightarrow P'a$

is not analytic (B 97). Now, of course, the part of this equivalence, which is the implication

L_8 $P'a \rightarrow \neg Pa$

must be valid, if we want to preserve the principle of non-contradiction with predicate negation, namely, the analyticity of the formula

L_3 $\neg (Pa \wedge P'a)$.

The only way in which it is possible to preserve L_3 and to deny the validity of L_7 is to say that the other implication which it contains, that is,

L_9 $\neg Pa \rightarrow P'a$

is not analytic. Since L_9 is equivalent to L_4, the latter must be abandoned by the same token.

[18] J. Vuillemin, *Identität in der Beweistheorie und die Kantische Fragestellung*, Kant-Studien 63 (1972), 289–304 (p. 301 n) is the only author I know of who has explicitly pointed to the fact that, according to Kant, antinomies imply the limitation of domain of applicability of the principle of excluded middle. He has, however, failed to distinguish between the excluded middle with propositional and with predicate negation and therefore to recognise that only the latter but not the former principle has thus been given a counter example by the antinomies.

Kant does not present his argument against L₉ as part of formal but as a part of his transcendental logic or a priori semantics. General or formal logic is right, he concedes, in not distinguishing affirmative from limitative or infinite predicative propositions. For this science abstracts from all content of the predicate, even though it be negative, and enquires only whether the predicate is ascribed to the subject or opposed to it, that is, it takes into account only the propositional negation. The formal logic must, however, be complemented by a priori semantics or transcendental logic. The latter, says Kant, also considers "what may be the worth or content of a logical affirmation that is thus made by means of a merley negative predicate, and what is thereby achieved in the way of addition to our total knowledge" (B 97). From this transcendental or a priori semantical point of view one is lead to the conclusion that the content and thus the truth conditions of " ¬ P" are different from those of "P'a".

Kant's argument, as presented in his comment of the table of judgments is the following: by a negative predicative proposition (¬ Pa) I do not make an affirmation but only "warn off" error of making one. By a limitative proposition (P'a) I *do* make an affirmation, namely, I locate the subject (a) in the unlimited sphere of non-P beings. Since the truth conditions of negative predicative and of limitative predicative propositions are thus different propositions the latter cannot be "passed over in a transcendental table of all movements of thought in judgements" (B 98).

The difference between negative and limitative predicative propositions as regards their objective truth conditions is taken into account by Kant himself in his comments on logical grounds of the first antinomy. According to Kant, antinomic propositions of both temporal and spacial antinomy are not like contradictory but rather like *contrary propositions* in the sense of the traditional logic. Traditional *iudicia contrarie opposita* are pairs of propositions such that one of them universally affirms the inclusion of a predicate (sphere) into another predicate (sphere) while the other denies it. As a consequence, each of them says more than what is required for a logical negation of the other. There is thus no contradictory opposition between them. As "falsity may lie in the excess content", both such propositions can be false, although not both can be true. In other words, whereas they do obey the law of non-contradiction, they fail to satisfy the law of the excluded middle. We can conclude from the truth of one of them the falsity of the other but not, conversely, from the falsity of the one the truth of the other. We cannot, accordingly, indirectly prove one of them by falsifying the other or by reducing it to absurdity.

The same is true of propositions of the antinomy. Kant shows it by considering the following formulation of the spatial antinomy, equivalent to the initial one: The world is infinite in extension or it is not infinite *(non est infinitus)* but is finite (non-infinite) (B 531-2). In our symbolism

R₁₀ $F'w_s \lor (\neg F'w_s \land (Fw_s \leftrightarrow F''w_s))$

This disjunction is not analytic, says Kant, because the antithesis *says something more* than is required for a simple negation of the thesis (B 532). In other words, we have got here a conflict analogous to the traditional opposition between contraries.

Now, the antithesis in this example is a conjunction and the first conjunct of it, "$\neg\, F'w_s$", is clearly the contradictory opposite of "$F'w_s$". As Kant explains, if we say "that the world is either infinite in extension or is not infinite *(non est infinitus)*, and if the former proposition is false, its contradictory opposite, that is the world is not infinite, must be true" (ibid.). The disjunction "$F'w_s \vee \neg\, F'w_s$" is valid and L_2 applies.

The excess content in this case must therefore be attributable to the second conjunct of the present antithesis, namely, to "$Fw_s \leftrightarrow F''w_s$". Kant shows it in the following way. If we say, he argues, no more than that the world is not infinite ($\neg\, F'w_s$) we deny the existence of an infinite sensible world *without however affirming in its place a finite world* (B 531). On the other hand, if we say that the world is finite or non-infinite ($Fw_s \leftrightarrow F''w_s$), we do not only remove the (actual) infinitude but attach a new determination to the world, namely, its (actual) finitude (B 532). The first proposition ($\neg\, F'w_s$) is therefore the contradictory opposite of the first disjunct of R_{10} ($F'w_s$), whereas the second ($Fw_s \leftrightarrow F''w_s$) is not because it has an excess content. In another text (*Fortschritte* A 95, 192 / Ak XX, 291, 328) Kant points out that the same is true of the temporal antinomy as well: it consists of propositions which are like traditional *contrary* propositions.

The point Kant is making is that the disjunction which according to the dogmatic realist formulates the first cosmological problem is not analytic at all on grounds of a semantical analysis of truth conditions of propositions employed to state it. It seems that we have here a case in favour of Kant's suggestion of B 98 that "the function of the understanding" expressed by limitative judgments "may perhaps be of importance in the field of its pure a priori knowledge".

Later on in the first *Critique* Kant has given more thought to the difference in meaning between negative and limititative propositions. He points out that the logical and the objective or material content (extension) of a concept, are not one and the same thing. A concept is said to have a logical content if it is self-consistent, that is, if its notes obey the law of non-contradiction. Logical content of a non-contradictory concept can always be given a reference in pure intuition by means of spatial and temporal schemata, that is, by extensive quanta and their parts. I shall call the study of this procedure of providing concepts with reference *pure intuitive semantics*. However, self-consistency does not ensure that a concept is not empirically empty. Empirical or objective possibility of a concept is guaranteed only if there is an operation of synthesis in empirical intuition of an object to which this concept applies. The existence of such an operation cannot be proved only from the logical principles of analysis of concepts (like those of non-contradiction and of the excluded-middle). Other principles of possible experience, in particular, the synthetic principles of the understanding together with the rules for schematisation of concepts must also be considered (B 625 n). Kant's theory of reference as based upon the procedure of possible synthesis in empirical intuition may be called *material* or *empirical intuitive semantics*.

The semantical difference between "$\neg\, Pa$" and "$P'a$" can further be explained in terms of Kantian material semantics, that is, by considering the *objective* possibility of "P", "P'" and "a". It is plain that if "P" is an objectively possible predicate and "a" an

objectively possible individual and "¬ Pa" is true, "P'a" is not necessarily also true. That would be the case only if the objective possibility of "P" ensured by itself the objective possibility of "P'". This is not the case, as can be shown on the example of the predicate pair "actually finite" and "actually infinite" which appear in the first antinomy: while the first predicate is obviously objectively possible, the second one is not. An independent proof for "P'" is therefore still required, based on the *synthetic* principles of possible experience. This explains why in speaking about empirically possible objects we cannot conclude from the truth of "¬ Pa" to the truth of "P'a", and why L_4 is not a universally valid logical law.

VII. *Transcendental realism viewed as bad a priori semantics*

We have seen that R_5, which is falsified by the first antinomy, can also be derived from R_1, R_2, L_2 and definitions of finite and infinite magnitude. So, either R_1 or R_2 must be rejected, if L_2 is to be safe. This shows that L_4 is not the only root of the first antinomy. As a matter of fact, it is not even the deepest one. For, according to Kant, the application of L_4 in the present context presupposes, that is, is based, on R_1. He writes: "If we regard the two propositions, that the world is infinite in magnitude and that it is finite in magnitude, as contradictory opposites, we are assuming that the world, the complete series of appearances, is a thing in itself that remains even if I suspend the infinite regress in the series of its appearances" (B 532).

This explains why the falsity of R_5, which is a case of L_4, also falsifies R_1. If the sensible world is, as the transcendental realist maintains, a whole existing in itself, than, of necessity, it is either finite or infinite. But, argues Kant, "both alternatives are false (as shown in the proofs of the antithesis and thesis respectively). It is therefore also false that the world (the sum of all appearances) is a whole existing in itself" (B 534).

Recalling our reconstruction of steps leading to that formulation of the first cosmological problem from which the first antinomy can be derived, we can say that R_1, is disproved by the following argument:

(1) Indefinitely many extensive temporal and spatial quanta ($q_{t,s}$) are actually given in our experience.
(2) By the T-thesis (R_1), $q_{t,s}$ are things existing in themselves.
(3) By the transcendent version of the fundamental principle of pure reason (R_3), indefinitely many Kantian worlds ($w_{t,s}$) are also given and exist as things in themselves in objective time.
(4) By the I-thesis (R_2), these worlds can be consistently thought of and known by means of non-schematised categories, including the category of quantity.
(5) Consequently, Kantian worlds are necessarily either actually finite or actually infinite as regards time and space ($Fw_{t,s} \vee F'w_{t,s}$).
(6) By the proofs of the first antinomy Kantian worlds are neither actually finite nor actually infinite ($\neg (Fw_{t,s} \vee F'w_{t,s})$).
(7) So, Kantian worlds cannot be consistently thought of nor *a fortiori* known in non-schematised categories (refutation of R_2).
(8) Also, they do not exist as things in themselves (refutation of R_1).

It is easy to see, why, from the point of view of a priori semantics, the disjunction above (5) must be false. Since the world is sensible, it must be given in time, that is, in the empirical synthesis of the series of appearances and, since this series is always conditioned, it can never be given as complete. Therefore it cannot possibly exist as finite or infinite magnitude either (B 533). If we say that the world is actually *finite* as regards space and time, we attribute to it a measure which is *too small,* for it is given by empirical regress *in indefinitum.* And, if we say that it is actually *infinite,* it becomes *too large* in view of the same condition of its givenness. Once again, the logical division of magnitudes in actually finite and actually infinite, which is unobjectionable in the domain of magnitudes in themselves does not hold in the domain of sensible magnitudes. In this domain, the above division becomes demonstrably paradoxical. With respect to the sensible world one cannot consistently hold the T-thesis and the I-thesis. That is, one cannot *consistently* suppose that the sensible world is a thing in itself and think of it by means of categories, for instance, by applying to it the category of quantity, as the transcendental realist does when he formulates the first cosmological problem. Thus, the sensible world is either not intelligible to us (the consistent applicability of categories to an object is indeed the very condition of its intelligibility) or it is not a thing in itself.

The non-analyticity of "$Fw_{t,s} \vee F'w_{t,s}$" is a case against the I-thesis, according to which appearances and the sensible world, taken as existing in themselves, can consistently be represented and eventually known through propositions which employ non-schematised concepts. Which kind of error is contained in the I-thesis? Being an error concerning the consistency and thus the truth condition of antinomic propositions, it is clearly an error in *semantics* of these propositions. The I-thesis itself (R_2) can therefore be viewed as a piece of bad a priori semantics. Notice, that the first antinomy does not refute the assumption that things in themselves can be, firstly, consistently represented and secondly, known in propositions employing non-schematised categories. The first possibility is, as said above, explicitly admitted by transcendental idealism. The second possibility is not, yet its falsity is not followed from the antinomies themselves but from a critique of proof methods in metaphysics in general.

Since the thesis that the sensible world is a thing in itself leads to contradictions, it is itself not only false but also self-contradictory. Kant brings this point out by saying that the concept of sensible world existing in itself is self-contradictory in the same sense in which the concept of four-cornered circle is self-contradictory (*Prolegomena* § 53, cf. *Fortschritte* A 192/Ak XX, 328). The reason why the antinomic propositions of the first antinomy are false is therefore the same reason for which propositions (1) A four-cornered circle is four-cornered and (2) A four-cornered circle is round are false: they all employ inconsistent concepts, the former that of "sensible world existing in itself", and the latter that of "four-cornered circle". The self-contradictory character of the second concept can easily be followed from definitions of "circle" and "four-cornered". The same is not true of the concept of sensible world existing in itself. At its face value that concept seems quite consistent and has actually been treated as such by antique as well as by modern metaphysicians (up to Strawson). Kant himself does not seem to

have discovered the inconsistency of that concept directly by analysis of its notes but from contradictions to which it gives rise. Only in that indirect way did he gain the insight that "appearances (as mere representations) and their complexes (such as the sensible world) which yet are to be given *in themselves* (as objects) are something impossible" (B 821). Appearances existing in themselves are something impossible because to say that they are given or exist in themselves is the same as to say that there is experience previous to any experience, which is absurd (*Prolegomena* § 52 c). This, reason, however, does not seem to have been the *ratio cognoscendi* but rather a later explanation of the error contained in the T-thesis and discovered previously by the antinomies.

It is a view of Kant's that concepts of objects in general can be logically divided into concepts of possibility (of something) and concepts of impossibility (of nothing), (B 346). A concept which contradicts itself is among the latter: it is not said to be without object but to refer to a kind of impossibility or kind of nothing. For this reason it is also called an "impossible concept" (B 820, *Prolegomena* § 52 b).The "nothing" which such a concept may be employed to name is an "empty object without possible concept" or *nihil negativum* (B 348). In agreement with this view, Kant can say that the expression "a four-cornered circle" *names (nennt)* an object although by the concept "four-cornered circle" no *thought* about it is expressed (*Prolegomena* § 52 b). Assertions "based on impossible concept of the object" are not just non-sensical but false. In this case, Kant explains, "we can apply the rule: *non entis nulla sunt predicata.*" In agreement with this, "all that is asserted of the object, whether affirmatively or negatively, is erroneous, and consequently we cannot arrive apagogically at knowledge of the truth through refutation of the opposite" (B 820-1).

We see that Kant treats definite descriptions like "the sensible world existing in itself" as referential expressions. As a consequence, Kant's views concerning the antinomic character of propositions of the first antinomy cannot be adequately brought out by giving definite descriptions of the existential analysis *à la* Russell. According to Russell, a sentence of subject-predicate form which contains a definite description is either meaningless or we must suppose that it is about something. This means that the proposition "The sensible world existing in itself is finite (infinite)" is either meaningless or there is a sensible world existing in itself for it to be about. According to Kant, there is nothing which this sentence is about and it is not just meaningless but false.

I have said that the T-thesis boils down to an error concerning the reference of the definite description "the sensible world existing in itself". It consists in thinking, that this description refers to a possibility whereas its object is demonstrably an impossibility. How is such a mistake about the constitution of referents of cosmological ideas produced? Kant's answer is: by the error of subreption attributable to our faculty of judgment (B 671). *Subreption* in the Kantian sense[19] generates illusions about the nature

---

[19] For the traditional sense of "subreption" cf. W. T. Krug, *Allgemeines Handwörterbuch der Philosophischen Wissenschaften.* Leipzig: Brockhaus, 1829.

of referents of our concepts by presenting subjective conditions of our intuition and thought as "determinations of things in themselves" (B 353, cf. B 819). In other words, by positing formal principles as constitutive. A typical example of such substitution is provided by the T-thesis: it projects subjective formal conditions of our sensible intuition, which are time and space, over empirical things and thus produces the illusion that appearances exist in objective time and space, that is, in themselves (cf. A 389). Obviously, Kantian subreptions are errors of the *determinating judgment*, that is, of *application* of a priori given general forms to empirical data. As our a priori knowledge that and how a priori intuitions and concepts relate or refer to objects is called transcendental (B 80), the corresponding a priori errors of subreption are also called transcendental (B 611, 647). The transcendental realist illusion about the true nature of things as objects of senses (B 535) is thus reduced to an error of application of subjective forms of intuition. In other words, the transcendental realist is proposing bad metaphysics because his a priori semantics of time and space concepts is erroneous. Kant himself points out that semantical considerations are the best means for laying bare the illusion expressed in the T-thesis. What sets us on the right path is the suspicion "that the cosmological ideas, and with them all the mutually conflicting pseudorational assertions, may perhaps rest on an empty and merely fictitious concept of that in which object of these ideas is given to us" (B 518), which seems indeed, to be a suspicion about an error concerning reference.

We can, finally, ask whether the transcendent version of the fundamental principle of pure reason (R_3) also contributes, along with R_1 and R_2, to generate this "empty and fictitious" conception about the mode of givenness of referents of cosmological ideas. The answer is that it does. In order to see this we must recall that R_3 allows us to follow the givenness or existence of the unconditioned from the givenness or existence of the conditioned, the mode of givenness and of existence in both cases being the same. Combined with the T-thesis, this principle implies that the sensible world exists in itself and leads thus to the antinomies. It might seem that R_3 is unobjectionable in itself and that the whole trouble springs from combining it with the T-thesis, which is false. This, however, would not be a correct rendering of Kant's views of the matter. Kant concedes, no doubt, that R_3 can legitimately be employed with things in themselves as when he notices that "... *if the conditioned as well as its condition are things in themselves*, then upon the former being given, the regress to the latter is not only *set as a task*, but therewith already *given*. And since this holds of all members of the series, the complete series of the conditions, and therefore the unconditioned, is given therewith, or rather is presupposed in view of the fact that the conditioned, which is only possible through the complete series, is given" (B 526, the first italics are mine).

Kant warns, however, against forgetting that the synthesis of the conditioned with the unconditioned asserted in this passage is not empirical nor any other intuitive one, but conceptual or intellectual: "But synthesis of the conditioned with its condition is here a synthesis of the mere understanding, which represents things as they are, without considering *whether and how we can obtain knowledge of them*" (B 526-7; last italics are mine).

It could not be otherwise, because both the conditioned and the unconditioned are things in themselves given only abstractly in pure thought. This explains why such *transcendent employment* of the fundamental postulate is legitimate. When in this case we infer the givenness of the absolute totalities of conditions from the givenness of the conditioned, we need not presuppose more than what is already assumed by the logical requirement upon which the fundamental postulate in its original form is based, namely, that we should have adequate premises for any item of knowledge.

Whereas the transcendent employment of the transcendent version of the fundamental principle of pure reason is thus safe from objections, its *empirical employment* is unsound. For it leads to the conclusion that the mode of givenness and of existence of conditioned empirical objects (appearances) and of their unconditioned is the same and that, accordingly, absolute wholes or totalities of conditions are given or exist in the same sense as empirical and relative ones, which as we know from antinomies, is erroneous. We cannot legitimately infer, *taking the term "given" in the same sense,* that if a conditioned appearance is given, "all its conditions (as appearances) are likewise given" (B 527). We cannot do so because it is obviously false to say that "if the conditioned, in the [field of] appearance, is given, the synthesis which constitutes its empirical condition is given therewith and is presupposed" (B 527). Appearances are given in empirical synthesis of apprehension and only in it (B 527). Yet, the performance of this synthesis does not presuppose the performance of the synthesis in which all empirical conditions of appearances are given. The latter, says Kant, "first occurs in the regress, and never exists without it" (ibid.). Whereas, thus, in reasoning about things in themselves, we can conclude from the givenness (existence) of the conditioned to the givenness (existence) of the absolute totality of its conditions, in reasoning about appearances we cannot.

We see that the employment of R_3 with empirical conditioned is unsound quite independently of the T-thesis, and that this principle plays a positive role in generating the dialectical illusion which leads to the first antinomy. What the T-thesis does is to reenforce the original illusion that R_3 is applicable in reasoning about appearances. For the T-thesis asserts that appearances are things in themselves and R_3 is legitimately applied, as we have just seen, to such kind of conditioned entities.

We have now reached the *main source* of all cosmological antinomies: they originate from the same principle from which cosmological problems themselves are generated, namely, form the fundamental principle of pure reason, taken, however, in the transcendent version, peculiar to transcendental realism. By applying this version to empirically given temporal and spatial quanta taken as things in themselves we necessarily generate a cosmological problem about the magnitude of the absolute totality of conditions of these quanta, namely, of the sensible world existing in itself. Since the sensible world, the sole datum of the first cosmological problem possesses demonstrably paradoxical properties, one comes to the suspicion that antinomies may perhaps be due to *errors in the very formulation of the problem.* This is the course taken by Kant himself. As we have seen, he has identified several such errors. The central error of the conflict represented by the antinomies, however, is "due to an illusion

which arises *from our applying to appearances* that exist only in our representations, and therefore, so far as they form a series, not otherwise than in a successive regress, that idea of absolute totality which holds only as a condition of *things in themselves* (B 534, italics mine).

The fundamental mistake of transcendental realism in formulating the first cosmological problem lies therefore in the failure to distinguish between the T- and the P-mode of existence and, accordingly, between the domain D_t of things existing in themselves and the domain D_p of empirical things. This mistake can conveniently be called error of *referential monism*. To it transcendental realism adds another one which consists in thinking that D_p is properly contained in D_t (T-thesis). The two domains are however different. They are different, in the first place, because their members are given through different constitution procedures. The members of D_t are constituted in pure thought by means of non-schematised concepts in agreement with formal logical principles. The elements of D_p, on the other hand, are constituted in experience in agreement with synthetic principles of possible experience as well as with formal logical principles (B 625 n). Once this difference in constitution procedures is noticed, one easily sees that results unproblematic in one domain are not necessarily so in the other. As the preceeding analysis has shown, the concept of absolute totality does not apply in the two domains in the same way, and the logical division of magnitudes in actually finite and infinite, while unobjectionable in D_t breaks down in D_p. The illusion which consists in thinking of things in terms of referential monism is, however, quite natural and, were it not for the antinomies, we could scarcely be able to uncover it. The definite cure from it can of course be obtained only within the system of critical philosophy.

VIII. *Kant's Way Out of the First Antinomy*

The error of transcendental realism (and of common sense) is to take appearances as being things in themselves describable by means of empirically uninterpreted categories and ideas. In so far as it does not pay attention to the question of possible synthesis in experience of referents of concepts, transcendental realism neglects entirely the problem of givenness of objects of cosmological ideas and misconstrues, therefore, the first cosmological problem as being about things in themselves. Transcendental realism is not able to distinguish between the logical and the objective possibility of these concepts either. It therefore cannot but consider that pure formal negation is enough to constitute an objectively possible (derived) concept out of any given objectively possible category or idea. Yet, as we have seen, in the case of cosmological ideas, the logical predicate negation is not sufficient to guarantee the objective reality of the complement of "finite". We have here a typical case of *formal dialectical error*, due to the confusion between operation of constitution of a complement in the domain of concepts without content and in the domain of objectively valid concepts. In the first domain the constitution of a complement is indeed identical to logical negation of a

predicate. In the second domain this operation is not sufficient for determing the sphere of the complement. Accordingly, although we are allowed to generate all kinds of predicates of purely intelligible objects by means of predicate negation, these predicates cannot be applied to empirical objects as long as they are not provided with synthetic procedures for constituting their references. In modern terms, predicates applicable to empirical objects must be *decidable* in principle. The difference between logical and objective possibility of a concept must therefore be introduced, the price to pay being the rejection of the principles of *transcendental* or *a priori sementical* realism.

The fact that not all concepts are decidable in principle is thus the main positive reason of Kant's for rejecting L_4. It also explains the procedure followed by Kant *in his own formulation* of the first cosmological problem. The problem is about the magnitude of sensible worlds, whether it is finite or not. Kant starts therefore by making sure that the cosmological concepts involved in its formulation are *objectively possible*. This is achieved by specifiying the way of providing them with referents in possible experience, this way being the empirical regress *in indefinitum*. In accordance with this, objects of all concepts envolved in the Kantian formulation of the cosmological problem, ("sensible world", "finite" and "infinite") must be viewed as given in a synthesis of this kind.

There is, according to Kant, a noteworthy relation between the logical postulate of pure reason and the empirical regress. As regards extensive quanta, this postulate asks us, as said above, to find the series of all objective conditions or limits of each of them. And we know that these series must be given in empirical regress. The first cosmological problem reduces thus to the task of performing empirical regress. It is indeed analytic, according to Kand, that "if the conditioned is given, a regress in the series of its conditions is *set us as a task*" (B 526). Which means that the fundamental postulates of pure speculative reason function actually as a rule for empirical regress, giving the answer to the question "how we are to carry out the empirical regress, and how far we should continue it" (B 542). This answer is: *ad indefinitum*. The reason is the following. Limits which we can reach in the empirical regress must themselves always be regarded as empirically conditioned (B 546). An empirically unconditioned limit of extensive quanta, in particular, can only be the void, which, however, cannot possibly be encountered in a perception nor, *a fortiori* in an empirical regress. In view of the fundamental postulate the regress must therefore be continued at any stage actually reached. That is, we must go on to *enquire* for a still higher limit even though we may not be capable of finding it through experience (B 545). Such a regress may accordingly be said to go *in indefinitum*. It must be distinguished from the regress *in infinitum* where we necessarily find new members of the series of conditions, as in the case of regressive division of an empirical whole given in an intuition (B 541–2, 547).

The problem to find all objective conditions of an event requires thus the constitution of a series. This is obvious in so far as temporal conditions are concerned. For time is in itself a series and indeed the formal condition of all series (B 438). In regard to any event given at present, its conditions can be a priori distinguished from its consequents as the past is distinguished from the future. It is thus quite unproblematic to consider

temporal worlds as given by empirical regress in the direction of the past. In particular cases the regress can be conducted by means either of our own experience of the past, or of the guiding-thread of the history, or, also, of causal chains (B 549–50). On the other hand, it is not at first clear whether absolute totalities of conditions of spatial quanta can also be seen as given by regress. No doubt, the synthesis of parts of space, by means of which we apprehend space, is successive, "taking place in time and containing a series" (B 439). It cannot be otherwise, because spatial quanta are extensive magnitudes and we know from the Transcendental Analytic that extensive magnitude can only be represented and homogenous) parts (B 204; *Fortschritte* A 87 / Ak XX, 288), operation which is also called composition or measurement (B 439, 456 n). Yet, can this operation be viewed as regress? It seems that it cannot because the measuring of a space is a constitution of a series of conditions of a given conditioned in which "the side of the conditions is not in itself distinct from that of the conditioned" so that in space empirical regress and progress "would seem to be one and the same" (B 440). We should keep in mind, however, that spacial worlds are conceived as series of spatial quanta which limit, that is, enclose each other in order and that those quanta "which are thought in extension of the given space are always the condition of the limits of given space". Each space, in so far as it is enclosed in other space, must be considered, Kant points out, "as being also conditioned, in that it presupposes another space as condition of its limits, and so on" (ibid.). This answers our question: the measuring can be viewed as empirical regress in so far as it generates the series of limits of any given empirical spatial quantum.

After having established the correct formulation of the first cosmological problem, Kant goes on to solve it. He does it by proving the truth of the antithesis which says that the world has no beginning, and no limits in space, but is infinite as regards both time and space, in our formalism "$\neg\, Fw_{t,s} \wedge F'w_{t,s}$". Kant starts with an indirect proof of "$\neg\, Fw_{t,s}$", which consists in showing that "$Fw_{t,s}$" (the thesis) is absurd. If we suppose, he argues, that the world is finite as regards time and space, it would have to be limited by empty time and empty space, which would require us to have a perception of limitation by absolutely empty time and space. But such a perception, as completely empty of content, is impossible. So "$Fw_{t,s}$" is absurd. Therefore, concludes Kant, "$\neg\, Fw_{t,s}$" is true (B 549). This is, he says, the first and *negative* answer to the cosmological problem regarding the magnitude of the world (B 548). It is followed by a *positive* answer which affirms "that the regress in the series of appearances, as a determination of the magnitude of the world, proceeds *in indefinitum*" (B 549). This implies that the sensible world is indefinite as to its temporal and spatial extension, i. e., that "$F'w_{t,s}$"is true, where "F'" and "$w_{t,s}$" represent concepts constitutable in empirical regress *in indefinitum*.

A remarkable feature of this solution of Kant's is that the two parts of the antithesis are established *independently* of one another and that the problematic L_4 is not employed. In the first proof the *reductio ad absurdum* is employed, yet only to establish a negative conclusion. The whole argument differs therefore substantially from the dogmatic proof of the antithesis of the first antinomy produced in agreement

with principles of transcendental realism. As Kant stresses, in the latter "we regarded the sensible world, in agreement with the common and dogmatic view, as a thing in itself, in totality, prior to any regress". The same realist interpretation was given of course to all other cosmological concepts employed. For this a priori semantical reason the conclusion of the arguments was also different. The inference was not to the falsity of (actual) finitude but to "the actual infinity of the world" (B 549 n). Kant's own procedure is thus entirely in keeping with the rule advanced in the Transcendental Doctrine of Method that in solving metaphysical problems positive answers should always rest on direct or ostensive proofs and never on indirect or apagogical ones (B 817).