DECIDABILITY AND COGNITIVE SIGNIFICANCE IN CARNAP*

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ABSTRACT. Our interests in this paper are mainly historical. We give an analysis of the nature, origins, different formulations and liberalization of the Carnapian criterion of cognitive significance and we consider it as a chapter in the recent history of theories of problem solving by effective methods. We are not interested in appraising the internal merits of Carnapian effective methods. Only occasionally we venture into the current polemics about the criterion

INTRODUCTION.

[We find in Carnap first a strict criterion and later different liberalized criteria for cognitive significance.] The *strict*, constructivist criterion amounts to a set of general, direct and deterministic decision procedures for certain (infinite) classes of empirical questions. We call these procedures general because they (supposedly) give answers to all the members of the class of questions for which they are

(*) I am indebted to B. Barbosa Fº and H.R. Brown for many illuminating criticisms.

proposed, direct because they apply to questions themselves or to equivalent questions, deterministic because the questions answered are what-questions or yes-or-no-questions and not how-probable-questions. The strict criterion received alternatively semantic (phenomenalistic and physicalistic) and syntactic (linguistic, formal) formulations. These alternatives reveal Wittgensteinian (and later Tarskian) and Hilbertian influences, respectively. But the different formulations notwithstanding, Carnap's strict criterion never ceased to be a set of procedures of decision. It can safely be interpreted as an attempt to realize in the domain of empirical questions the old *rationalist* ideal of an universal effective method for solving problems.

The *liberalized* criteria, on the other hand, consist of partial or indirect or probabilistic decision procedures, equally for empirical questions. Conceptually and historically, the origins of the liberalized criteria are the negative solutions for decision problems for certain formal questions. It is only secondary that the liberalization is due to the impossibility of finding strict procedures for some important classes of questions in empirical science, such as questions about the applicability of disposition and theoretical terms.

Our interpretation is at variance with most current interpretations. The main difference is that none of them, to our knowledge, reconstructs the systematic connection that exists between the significance criterion and effective procedures. As a consequence, most of them are not consistent with the results of our interpretation.

1. FORMULATION OF THE STRICT CRITERION.

Carnap offered no single complete formulation of the strict criterion. Moreover, in no earlier writing did he give either the precise characterization of the descriptive language to which the criterion applies nor the complete list of linguistic entities to be classified as significant or non significant. The difficulty of the task of reconstructing the criterion itself is exacerbated by the fact that Carnap oscillated with regard to its precise formulation. In his first writings on the subject in 1928, he adopts semantic formulations, although in the early thirties he banishes semantic from logic and epistemology as beset

with 'insoluble difficulties and contradictions' (Carnap [6], p. 454), and in the late thirties, under the influence of Tarski, he switches back to it.

The reconstruction of the descriptive language that satisfies the strict criterion of cognitive significance becomes much easier if we take into account some later texts, in particular Carnap [8], [10] and [11]. The first of these texts contains a precise characterization of what Carnap considers to be a constructivist (or intuitionist or finitist) formal languages L_I. The purely logical part of L_I is an attempt at a formalization of the intuitionist requirements for a language adequate for constructing the intuitionist elementary arithmetic. And the purely descriptive part of L_I is intended to be used in the formal re-construction of empirically significant descriptive sentences. By relying on L_I we avoid unnecessary complications in early descriptive languages, such as those of the language of the AUFBAU based on the Russellian theory of types, without jeopardizing the exact comprehension of the significance criterion.

The logical part of L_I contains classical sentential connectives, variables for natural numbers, constants for number-theoretic properties, relations and functions, identity sign and a number of restricted operators, including limited sentential operators (quantifiers). The axioms of L_I are given by axiom schemata. The purely logical axioms consist of the axioms for the setential calculus, the restricted quantifiers and the identity. The arithmetic axioms characterize the K-operator and the successor relation. Besides explicit definitions and definitions in use, recursive definitions are also admitted (Carnap [8], Part I). The descriptive part of L_I obeys the same logical axioms as the logical part and satisfies, in addition, some other requirements, identical to the requirements for the observation language L_O stated in Carnap [10]. The variables of L_O take their values in (not necessarily infinite) domains of concrete empirical entities ('things' or 'total experiences') and every value of a variable is designated by an expression in L_O. All primitive descriptive constants in L_O are observational and all other constants reducible to the primitive ones either by explicit definitions or by definitions in use (Carnap [10], p. 41). The descriptive part of L_I so characterized is the language to which the strict significance criteri-

ion applies.

The guideline for reconstructing the significance criterion for such a language is given by some general semantic and syntactic properties of the *logical* part of L_I. In a late text Carnap mentions the following properties of this language: 'That language form provides a way to formulate unrestricted universal propositions about numbers, viz. with the help of open sentential formulas that are admitted as sentences. However, that language form cannot provide the formulation of unrestricted existencial propositions .... Each closed logical sentence S_i of that language is decidable, i.e. exactly one of S_i and $\neg S_i$ is provable and there is a (decision) procedure for discovering the proof. Each closed logical numerical expression A_i is computable, i.e. there is a procedure for discovering the numerical expression A_j in normal form ('0','0'', etc.) such that $A_i = A_j$ is provable' (Carnap [11], pp. 163-164). In order to be significant the descriptive part of L_I will be submitted to the same restrictions of its expression power and to the analogous requirements of constructivity of its sentences, predicates and functions. The Carnapian strict significance criterion is nothing else than a set of methods which assure that the descriptive part of L_I satisfies the constructivist requirements. However, it is clear that these methods cannot be the same as the decision procedures for the logical part of L_I. The decision problem for the question of significance of descriptive sentences, e.g. cannot, in general, be solved by formal proof procedures. The descriptive part of L_I is not axiomatized and, moreover, it is certainly not finitely axiomatizable. As an example of Carnap's views on this question, let us consider his analysis of the problem of deciding the truth values of the sentences about the relations of situation and the qualitative relations between given space-time points. This problem cannot be solved, observes Carnap, by a (formal) proof in L_I. 'A relation of situation in the simplest case will be expressed by means of an analytic (or contradictory) sentence (e.g. 'Positions 7 and 6 are neighbouring positions'). On the other hand, a qualitative relation in the simplest case, will be expressed by means of a synthetic descriptive sentence (e.g. 'Position 7 and position 6 have the same color'). The former sentence is determined by a logical operation, namely, a proof; the latter, on the other hand, can only be decided on the basis of empirical

observations, that is to say, by derivation from observation-sentence' (Carnap [8], p. 45). We see in this example some of the peculiarities of the decision problem for descriptive sentences. The positive solution of the problem implies the sharp division between logically provable and non logically provable sentences (the famous analytic - synthetic distinction) and is conditional on the existence of general decision procedures either by observation (the semantical, empirical, formulation) or by derivation (the syntactic, linguistic, formulation).

General solutions for the decision problems relative to different classes of expressions of the descriptive part of L_I are given by the Carnapian strict criterion of cognitive significance. To put it otherwise: the criterion in question amounts to a set of general, direct and deterministic decision procedures for certain (infinite) classes of empirical questions formulable in L_I. As we already said above, we call these procedures general, because they (are supposed to) give answers for all the members of the class of questions for which they are proposed, direct, because they apply to questions themselves or to equivalent questions, deterministic because the questions answered are yes-or-no-questions or what-questions and not how-probable-questions. Yet, as we also already noted, in no place did the strict criterion receive a complete and precise formulation. Moreover, it was the origin of many hesitations in the questions of detail. What we want to do now is to pick up what seem to be the most important statements of (parts of) this criterion, without limiting ourselves to the formulations relative to L_I. In different texts the criterion is applied to individual, function, predicate and relation constants, sentences and problems of descriptive languages (L_D) and even to the descriptive languages themselves. In most cases the criterion appears in both semantic and syntactic formulations.

1.1. *Syntactic criterion for individual constants.* A descriptive individual constants is significant if it is primitive in L_D or defined in it by a definition chain in which no unrestricted operators occurs.

This definition is a generalization for descriptive languages L_D of a definition given by Carnap in [8], p. 45, for L_I. Our procedure, employed also below, is justified by the fact that Carnap thinks it to be possible to give a purely syntactic characterization of the difference

between descriptive and logical expressions and that he considers all empirically significant languages to be equivalent (Carnap [6], p. 462)(1). Following the same source, we shall call a descriptive expression 'definite' if it is either primitive in a L_D or defined in it by a definition-chain in which no unrestricted operators occur. In a later text, Carnap declared that his term 'definite' is synonymous to the term 'effective' in the modern computability theory (Carnap [9], p. 193).

1.2. *Semantic formulation.* A descriptive individual constant is significant if it designates a concrete, observable entity that is or can be generated by constructive procedures from basic relation (s) among total experiences.

In the AUFBAU these constructive procedures are formulated in the so-called 'language of fictitious constructive operations'. The opertions considered by Carnap are supposed to be able to simulate the behaviour of human subjects, at least in certain cases of data processing, for instance, in the (hypothetical) case of the construction of quality spaces out of total experiences. Methodologically, the operation rules for the construction of objects (concepts) are considered necessary in order to assure univocity, non emptiness and extensional character of terms defined in the formal language of the AUFBAU. This requirement shows that in 1928 Carnap still did not have a clear idea of the syntax of a formal language that would satisfy the requirements of constructivism.

2.1. *Syntactic criterion for function constants.* A descriptive function constant is significant if it is definite, i.e., computable (Carnap [8], p. 45, and [9], p. 193).

In the AUFBAU the descriptive functors are defined by finite lists of significant descriptive individual constants.

2.2. *Semantic formulation.* A descriptive function constant is significant if it designates a set of pairs of total experiences (or things).

This definition is a consequence of the relation theory used in the AUFBAU.

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(1) Some authors, like Neurath, believed that there was only one significant descriptive language.

3.1. *Syntactic criteria for property and relation constants.*

a. A descriptive property or relation constant is significant if it is definite (Carnap [8], p. 45).

b. A descriptive property or relation constant occurring in a given atomic sentence of L_D is significant if the protocol sentences of L_D (cf. definition 4.) *from which* the given atomic sentence is derivable have been stipulated (Carnap [5], p. 224).

3.2. *Semantic formulation.* A descriptive property or relation constant is significant if the property or relation it designates is decidable (Carnap [8], p. 161 and [9], p. 193)

4. *Semantic criteria for protocol sentences.*

a. Significant protocol sentences constitute a subset of the sentences of the language of the physical system. A sentence belongs to the language of the physical system if it correlates a certain numerical value (or interval or probability distribution of numerical values) of a state variable to each member of a set of space-time points. Protocol sentences in such a language differ from other sentences in two aspects. First, they do not refer to single space time points but the space-time regions. Second, they do not correlate specific values of state variables to these regions, but rather large classes of unknown values (Carnap [6], p. 458).

b. A sentence is a significant protocol sentence if it is about states of affairs that obtain among total experiences that constitute the fundamental relation (s) of the AUFBAU (Cf. Carnap [3], §§ 119 and 180).(2)

5.1. *Syntactic criteria for sentences*

a. A sentence in L_D is significant if it is definite, i.e. if all constants in it are definite and all variables are bounded by restricted quantifiers (Carnap [8], pp. 45-46).

b. A sentence in L_D is significant if it is derivable *from* a set of protocol sentences in L_D (Carnap [5], p. 224).

c. A sentence in L_D is significant if sentences of the protocol language in L_D are derivable *from it* and not necessarily vice-versa (Carnap [6], p. 440).

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(2) The term 'state of affairs' is used here and below in the sense of the TRACTATUS.

d. A sentence in L_D is significant if protocol sentences of the language of the physical system are derivable from it and not necessarily vice-versa (Carnap [6], p. 463).

5.2. *Semantic formulation.* A sentence is significant if it expresses a conceivable state of affairs whose existence or non existence is decidable by a set of operations based on precisely characterized possible experiences (Carnap [4], pp. 325-326).

We see that all significant sentences are verifiable or falsifiable by possible, i.e. theoretically conceivable experiences, and that, conversely, all sentences so decidable are significant. An important consequence of this definition is that we may know that a sentence is significant without knowing whether it is true or false (Carnap [4], p.324).

6. *Criterion of significance for problems.* A problem is significant if it is either a yes-or-no-problem concerning a significant sentence or a what-problem concerning a significant functor (Carnap [3], § 180).

It is clear that all Carnapian significant problems are effectively soluble either by purely formal or by empirical procedures. Let us emphasize that, according to Carnap, not all *important* human problems are significant in his sense. That means that Carnap is very far from identifying the objectives of his constructivist theory of science with the ambitions of the classical rationalism. On the contrary, he is explicit about the limitations of the scientific rationalism: 'The proud thesis that no question is in principle unsolvable for science agrees very well with the humble insight that, even after all questions have been answered, the problem which life poses for us has not yet been solved. The task of cognition is a definite, well-circumscribed, important task in life, and it can certainly be demanded that mankind should shape that aspect of life which can be shaped with the aid of knowledge by a determined application of this knowledge, that is, by using the methods of science. Even if modern movements frequently underestimate the importance of science for life, we do not wish to fall into the opposite error. Rather, we wish to admit clearly to ourselves, who are engaged in scientific work, that the mastery of life requires an effort to all our various powers; we should be wary of the shortsighted belief that the demands of life can all be met with the power of conceptual thinking alone.' These

words, put in the close paragraphe of the AUFBAU, should serve as a serious caveat to all those who are want to challenge Carnap as a defender of a totalitarian scientific reason.

7. *Criterion of significance for languages.*

a. A descriptive language is significant if it is definite, i.e. if all expressions in it are definite (Carnap [8], p. 46).

b. A descriptive language is significant if all closed expressions in it are definite (*ibid.*, § 15).

2. ORIGINS AND NATURE OF THE STRICT CRITERION.

Our review of the formulations of the strict, constructivist, Carnapian significance criterion for descriptive languages L_D makes it clear that this criterion is identical with a set of purely syntactic (formal) and empirical decision procedures and that the decision problems presumably soluble by these procedures have close parallels in the logical part of L_I which supposedly formalizes the elementary arithmetic of the intuitionists. In particular, the decision problems for the question of verifiability of all observation sentences and of computability of all empirical functions correspond to the decision problems for provability of all logical sentences and computability of all numerical expressions in L_I, respectively. We want now to make clear that this parallel is not a mere accident but a technical counterpart of some purely philosophical views of the foundations of mathematics and empirical sciences, common to Carnap and to other students of the problem of foundations.

The constructive properties of L_I, such as decidability of formulas, computability of numerical expressions, solubility of all number-theoretic problems formulable in it and so on, were intended by Carnap to be a formalization of the vague intuitive and philosophical requirements on the foundations of elementary arithmetic laid down by the Intuitionists (Carnap [8], p. 46). In the intuitionist writings, only constructive expressions were considered to be *justifiable* in a purely rational way, and that was meant to imply that non-constructive expressions were *meaningless*. Accordingly, a language that would fulfill the intuitionist requirements for rational justifiability of its expressions would also fulfill the intuitionist criterion of significance. Coming back to Carnap,

it is clear that the constructiveness assures the rational justifiability in L_I. On the other hand, Carnap himself made it clear that L_I was also significant in the intuitionist sense: 'The language form under discussion here [L_I] agrees with certain philosophical views sometimes called 'finitism' or 'constructivism'. According to these views it is the case e.g. that unrestricted existential quantifiers with respect to infinite domains give rise to meaningless sentences, and that predicates and functors are meaningful only if there is a fixed procedure by which their applicability in any concrete case can be decided'(Carnap [11], p. 164). It is easily seen now that Carnap's conditions on the significance for L_D given by his strict criterion are closely parallel to the formalized intuitionist significance criteria for matematical languages. And we already know that the basic logic for L_D is the same as the logic of the purely logical part of L_I. This shows that the concepts of justification and of significance associated with the construction of L_D are closely related to earlier intuitionist concepts.

It has often been said (by Hempel, for instance), that the Carnapian significance criterion reflected basic principles of contemporary empiricism. We see now that this thesis is not to be taken in a restrictive sense. Indeed, the Carnapian criterion also reflects the basic principles of contemporary constructivism. Moreover, there are some other important parallels between the Carnapian significance criterion and influential contemporary views on the foundations of mathematics. The insistence on the question of problem solving was a basic characteristic of various programmes in foundational research in continental Europe. Since 1900, Hilbert repeatedly made public his convictions that every mathematical problem was soluble, although not by a specifiable general method. In 1925, for instance, he wrote: 'As an example of the way in which fundamental questions can be treated I would like to choose the thesis that every mathematical problem can be solved. We are all convinced of that. After all, one of the things that attract us most when we apply ourselves to a mathematical problem is precisely that within us we always hear the call: here is the problem, search for the solution; you can find it by pure thought for in mathematics there is no *ignorabimus*. Now, to be sure, my proof theory cannot specify a general method for solving every mathematical problem; that does not

exist. But the demonstration that the assumption of the solvability of every mathematical system is consistent falls within the scope of our theory' (Hilbert [14], p. 384).] In his TRACTATUS (1921) Wittgenstein expresses similar optimism as to the problem solving capacity of human cognitive apparatus; 'If a question can be framed at all, it is also possible to answer it' (TRACTATUS, 6.5). Wittgenstein also offered what he believed to be final solutions of many kinds of fundamental problems such as the problem of formulating a constructive criterion of validity of all mathematical and logical sentences. However, his algorithms for validity apply neither to all classical mathematics (Wittgenstein was an intuitionist) nor to the full classical logic. In 1928, Hilbert formulated what can perhaps be considered the most ambitious objective for his proof theory: to solve the problem of validity in the full classical predicate calculus by finding a decision procedure for valid formulas. Hilbert declared this problem to be the central problem of the whole mathematical logic (Hilbert and Ackermann [15], § 11).

It seems to be safe to say that Hilbert's and Wittgenstein's optimism was dominant among the contemporary students of the foundational problems, despite some discordant voices, such as Brouwer's. In [2] Brouwer declared, indeed, that Hilbert's 'axiom of solubility' of every mathematical problem and its equivalent, the principle of the excluded middle, were both false. As for Carnap, he sided resolutely with the optimists. In the AUFBAU, proposing his first solutions to the decision problem for empirical questions, he wholeheartedly subscribed to the 'proud thesis of the omnipotence of *rational* science' expressed in the TRACTATUS (Carnap [3], § 183, our italics).

Not only is Carnap's insistence on problem solving related to contemporary studies in the foundations of mathematics but so obviously are his specific solutions. We explicated above Carnap's debt to Intuitionism in this respect. We want now to recall that the whole Carnapian programme of a syntax of the language of science and, in particular, his syntactic formulations of the significance criterion stand under direct influence of Hilbert's proof theory. The fact is well known and scarely deserves to be documented from texts. We mention only Feigl's report of a discussion with Carnap, at about 1928: 'Quick with labels for new conceptions (rather

than carrying them out in exact detail) I told Carnap that the syntax (he then still called it semantics', a term which he applied – following Tarski after 1934 or 1935 – in the later well established sense) he was thinking of as formulated in a metalanguage, amounted to a 'Hilbertization' of PRINCIPIA MATHEMATICA. He accepted, smilingly, my designation as essentially correct' (Feigl [12], p. XIV).(3)

As for the semantic formulations of the criterion, they stood under the direct influence of Wittgenstein's algorithm of validity.

Thus, both from a historical and a conceptual point of view, the strict significance criterion for descriptive languages has only superficially to do with Humean empiricism. It is such more appropriately traced back to the Leibnitzian *calculus ratiotinator* or to Cartesian general methods for solving geometrical problems. A direct evidence supporting this conclusion is Carnap's own approximation between his attempt to apply the modern theory of relations to the task of analysing empirical reality (a task which included the formulation of the significance criterion) and some Leibnitz' projects: 'The fundamental concepts of the theory of relations are found as far back as Leibnitz' ideas of a *mathesis universalis* and an *ars combinatoria*. The application of the theory of relations to the formulation of a constructional system is closely related to Leibnitz' idea of a *characteristica universalis* and of a *sciencia generalis*' (Carnap [3], §3). There seems to be no doubt, indeed, that what Carnap actually tried to do by formulating his strict significance criterion was to realize in the domain of empirical questions the old rationalist ideal of solving problems by mere calculation.

In support of this conclusion let us examine just one difficulty in any interpretation of the strict significance criterion framed exclusively in terms of the *traditional* empiricism. The expression 'conceivable experience' used in semantic formulations is highly theoretical. Indeed, it is analytic that 'conceivable experience' is not a definite descriptive property constant. Again, it is analytically not a significant disposition term. In 1956 Carnap is quite clear about that this: that conceivable experience has been always understood by him and by Reichenbach

(3) The main target of the Hilbertization of the PRINCIPIA MATHEMATICA were, of course, the axioms of infinity and of reducibility.

as an experience compatible with the fundamental laws of physics and not dependent upon our empirical knowledge of observers' abilities (Carnap [10], pp. 53-4). Accordingly, Carnap's criterion pressuposes a highly theoretical account of possible experiences (never undertaken by anybody, by the way) and not mere introspection or, still less, experimental analysis of immediate sense data. We also note that the recognition of the impossibility of relying exclusively on introspection and constructive method, i.e., on the traditional empiricism plus modern deductive logic is implicit already in the AUFBAU, where Carnap uses abstract geometry and physical analogies in constructing properties of phenomenal objects.

By these remarks I do not want to imply that Carnap did not say that his significance criterion was an empiricist one. He certainly did. What I do want to say is that he could as well have called his criterion a rationalist or a formalist criterion of empirical significance and that *we* can safely do so. The strict criterion and other of Carnap's contributions to contemporary philosophy of science are not to be credited to either of the two sides in the *old* epistemological and theoretical conflict between rationalism and empiricism but rather considered as an effort to produce a synthesis of sound elements belonging to both of these traditional doctrines (Cf. Carnap [3], p. VI).

We have seen that Carnap offered both semantic and syntactic formulations of the strict significance criterion. He never tried, however, to show that these alternative formulations characterized *equivalent* descriptive language as significant. In fact, this is not the case, as is easily seen by comparing, for instance, the operations involved in the decision procedures in the two kinds of formulations. But Carnap did not always consider them even as *admissible alternatives*. In his first writings on the subject in 1928, he proposed only semantic formulations and contrasted the genetic method employed in defining concepts (objects) with non constructive definitions by formal axiom systems like the one used by Hilbert in FOUNDATIONS OF GEOMETRY (1899). The semantic genetic method characterizes (defines) only single definite objects (concepts), as illustrated by Dedekind's cuts, by inductive definition of natural numbers and, of course, by Carnap's own constructive definitions of empirical (phenomenal) objects in the AUFBAU. In contradistinction, the

formal, syntactic method of implicit definitions by a (syntactically consistent) set of uninterpreted axioms, such as Hilbertian ones, laid down at the outset, defines only classes of objects. But at that time, Carnap considered classes as 'improper' or 'quasi-objects', in close agreement with Russel's 'no class theory'. And whereas the genetic method of introducing objects guaranteed the definiteness of all objects introduced, the axiomatic method did not in general assure the univocity (categoricity) of the object systems (models). After 1928, remaining as we have seen, under direct influence of Hilbert's formal proof theory,Carnap became reconciled with the formal axiomatic method and switched to syntactic formulations of his strict criterion. He went even so far as to identify logic with formal syntax (metamathematics): 'We have already seen that this formal method of general syntax can also represent [concepts which are sometimes regarded as not formal and designed as] *concepts of meaning* (or concepts of a logic of meaning), such as, for instance, consequence relation, content, relations of contents, and so on. Finally, we have established the fact that even the questions that refer to the interpretation of a language, can be handled within the domain of a formal syntax. Accordingly, we must acknowledge that all questions of logic (taking this word in a very wide sense, but excluding all empirical and therewith all psychological reference) belong to syntax' (Carnap [8], p. 233). As we shall show below Carnap not only believed that semantics was dispensable in methodological and logical questions but also that it led to 'unsoluble difficulties and contradictions'. Even after Gödel's theorems of incompleteness and of improvability of the consistency of the elementary number theory by finitary syntactic methods, Carnap continued to attempt syntactic formulations of this significance criterion, presented hereafter in liberalized forms and characterizing non-constructive languages. After the Paris Conference on the Philosophy of Science in 1935, under the strong influence of Tarski, Carnap returns to semantic formulations and treats *his* problem of meaning as 'belonging to the field which Tarski calls *semantics*', a theory of 'the relations between expressions of a language and things, properties, facts, etc., described in the language' (Carnap [7], p. 73). The languages here considered were not exclusively constructive languages. The Carnapian significance criteria written in Tarskian key were already

liberalized.

This outline of the changes in the formulations of the Carnapian significance criterion shows clearly that these changes are related in an essential way to some general philosophical questions and to developments in basic fields of logic and mathematics and not to peculiarities and eventual advances of philosophical empiricism or of empirical psychology.

Let us now take a closer look at two 'insoluble difficulties' in the use of semantics in the foundations of logic and methodology, and consequently in the formulation of the significance criterion. Both difficulties concern the empirical semantics developed according to the model of the TRACTATUS.

We have seen that, according to the semantic formulation of the strict significance criterion, a sentence of L_D is significant if it expresses a state of affairs whose existence or non-existence are decidable by conceivable experiences. According to the phenomenalist view of the AUFBAU an empirically decidable state of affairs is a structure (configuration) of immediately given total experiences (vertical slices of the solipsistic time-flow) or, more precisely and formally, list of lists of numbers taken from an enumeration of such data. Now, the immediately given total experiences are generally not common to different human subjects. Accordingly, the semantic formulation of the strict criterion is essentially solipsistic: it does not guarantee inter-subjective public significance for descriptive sentences (Carnap [6], p. 454). Under Neurath's influence (Neurath [17]), Carnap made the proposal that even specifying the significance of a given sentence, we should stop speaking about 'sense data' and the comparison between the sentence and the 'reality' and instead limit ourselves to the analysis of syntactic derivability relations between the sentence and the class of protocol sentences, whose vocabulary, formation and transformation fules would have to be syntactically characterized.

A second difficulty with empirical semantics is that it does not allow for a general definition of logical consequence. In order to be able to show why Carnap thinks so, we must recall Wittgenstein's definition of logical consequence. Let us call the n-tuples of truth-values of atomic sentences occurring in a truth-functionally composed molecular sentence which make that sentence true, the 'truth-grounds' of that

sentence (TRACTATUS 5.101). The concept of logical consequence is now defined as follows: 'If all truth-grounds that are common to a number of propositions are, at the same time truth-grounds of a certain proposition, then we say that the truth of that proposition follows from the truth of the others' (TRACTATUS 5.11). Wittgenstein himself adds the following comments on this definition. If q follows from p, then the truth-grounds of q are contained in the truth-grounds of p. Also, the sense of q is contained in the sense of p. But the sense of a sentence is the state of affairs it describes and affirms or negates (TRACTATUS, 4.064, 4.2). Thus, if q follows p, the state of affairs described by q is contained in the state of affairs described by p. Carnap now observes that in such semantics, no universal logical consequence concept can be defined, and in particular, not the one needed for logically connecting sentences describing phenomenal and physical states of affairs. He argues in the following way. The sentences of the first kind mentioned describe structures of solipsistic total experiences, and those of the second kind configurations of physical objects, such as electrons, electromagnetic fields, etc. But the states of affairs described in the first case and in the second case belong to completely separate material spheres of objects. Accordingly, they cannot be contained in one another. Now, suppose that the logical implication relation exists between a sentence of one of the two kinds and a sentence of the other kind. Then by the Wittgensteinian semantics, the state of affairs described by the implied sentence is a part of the state of affairs described by the implying sentence. But this is impossible. Consequently, empirical semantics based on the semantics of the TRACTATUS is inadequate as a foundation of the logic of the descriptive languages which contain phenomenalistic and physical constants (Carnap [6], p. 453). Carnap's way out was the general syntax.

This analysis shows that in early thirties Carnap was still far from the idea of interpretation of descriptive terms over abstract domains which themselves admitted empirical interpretations, i.e. from Tarskian or formal semantics. It furnishes also an interesting consequence for the interpretation of Carnap's contemporary ideas about the structure of descriptive theories. As far as the semantic formulation of the strict significance criterion of the AUFBAU is concerned, no physical sentence,

in particular, no theoretical sentence of a physical theory can be reduced to phenomenalistic sentences. More drastically, to require it would be semantically absurd. This result is contrary to many influential reconstructions of the early Carnapian ideas on the subject. Our contention is that the reductionist tendencies of early Carnap were greatly exagerated by later commentators. But to show that in details is beyond the scope of the present paper.

We want, finally, to point out that the strict criterion is closely related to the Carnapian thesis of extensionality. This thesis says that in every statement about a concept, this concept may be taken extensionally, i.e. it may be represented by its class or relation extension (Carnap [3], § 43). As applied to the constants of L_D the strict significance criterion is nothing else than a method of decision for the extensions of the concepts designated by the constants. This remark is particularly important for appreciating some standard objections to the criterion. Popper, for instance, considers it to be an obvious *absurdity* to use verifiability as a meaning criterion for sentences, because, he argues, it is necessary to 'understand' a sentence in order to be able to judge whether or not it could be verified (Popper [18], p. 63). It is easily seen that this objection, in order to apply to Carnap at all, must be so interpreted as to imply either that the Carnapian extensionally thesis is an outright nonsense or that it is absurd to require that the extensions be decidable. But such claims does not seem to be very obvious, indeed. In a similar vein, Marhenke [16] argues that Carnap's significance criterion for sentences is *false* because one can not know that something is evidence for or against a sentence unless one knows what that sentence means. But Marhenke misses to note that his objection presuposes the thesis that (at least) empirical sentences are intensional. Accordingly, he spends no effort in establishing this intensionality thesis.

3. LIBERALIZATION OF THE STRICT CRITERION.

After rejecting semantics from the foundations of logic and epistemology, Carnap was left with syntactic formulations of the strict criterion only. In these formulations he used syntactic terms like 'prova-

bility', 'derivability', 'computability' and 'definability'. The use of the criterion was justified by the belief that the decision problems for applicability of those terms in exhaustively characterizing large classes of descriptive and logical languages were or could be given positive solutions. The early Carnap was, in particular, optimistic, as we noted above, about the possibility of giving a complete and possibly also purely constructive criterion of validity of the sentences and of significance of the expressions of the *classical* logic and mathematics. Soon, after Carnap's adoption of this view (in about 1928, Godel's limitation theorems dealt it a terrific blow. Carnap's monumental LOGICAL ANALYSIS OF LANGUAGE, completed in 1933, is a long meditation on the possible way out. The solution found was the liberalization of constructive requirements.

We are not going to discuss the substitute for the constructive criterion of validity. We only mention that Carnap explicitely gave up the search for a constructive criterion of validity of the whole of mathematics as well as the Hilbertian program of constructing a *complete* formal criterion of validity, which, although not itself definite (effective) would still be based on definite rules (Carnap [8], pp. 99-100). As for the problem of cognitive significance, the strict significance criterion was substituted by a liberalized one which admitted the use of indefinite syntax languages (metalanguages) and of indefinite logical and descriptive object-languages. The thesis of the equivalence of the significant descriptive languages was abandoned. The principle of tolerance was announced: Everyone is at liberty to build up his own form of language; all that is required in doing so is to clearly state the syntax of the proposed language.

The use of indefinite syntax languages, conditioned only by the informal principle of non-contradiction, is now justified exclusively by methodological considerations and no more by stronger epistemological views such as those of constructivism. No question of justification or of correctness is raised. The terms 'analytic' and 'contradictory', for instance, have been proved indefinite in most relevant formal systems (Carnap [8], p. 165). Carnap uses them in his syntax because they are useful 'from the standpoint of certain general considerations' (*ibid.*, p. 175). One important theoretical advantage of these terms consists in

the fact that 'with their help the complete division of sentences without descriptive constants into analytic and contradictory is possible', whereas the corresponding classification of the same class of sentences into provable and refutable ['provable' and 'refutable' being also indefinite] is incomplete as a consequence of Gödel theorem (*ibid.*, p. 173). Another methodological virtue of these terms is that they are 'more closely connected with material interpretation of language'. (*ibid.*, p. 175).

In a quite analogous way, Carnap now admits *indefinite* mathematical terms. From the technical point of view this amounts to the admission of definition-chains where unlimited quantifiers occur. A definition of the form

$$P_1(x) = (\exists y)\ (Q(x,y))$$

where 'Q' is a definite number-theoretic relation constant and 'P_1' a new constant, are now tolerated, although the question whether, for instance, '$P_1(5)$' is true or not is not answerable by a definite method. To the standard intuitionist objection that '$P_1(5)$' is a meaningless formula Carnap now replies: '... it is true that we know of no method of searching for the answer, but we do know what form the discovery of the answer would be – that is to say, we know under what conditions we would say that the answer had been found. This would be the case, for example, if we discovered a proof of which the last sentence was '$P_1(5)$'; and the question whether a given series of sentences is a proof of this kind or not is a definite question. Thus there exists the *possibility of the discovery of an answer*, and there appears to be no cogent reason for rejecting the question' (*ibid.*, p. 161). Carnap justifies the use of *impredicative* constants in just the same way. He sums up his position on admissibility of meaningfulness of mathematical constants as follows: 'In general, since there are sentences with unrestricted operators which are demonstrable, there is always the possibility of coming to a decision as to whether or not a certain indefinite or impredicative term is applicable in a particular individual case, even though we may not always have a method at hand for arriving at this decision. Hence such terms are justified even from the standpoint which makes the admissibility of any term dependent on the possibility of a decision in every

individual case. (Incidentally, in my opinion, this condition is to narrow, and is not convincingly established)' (*ibid.*, p. 164). The parenthetical remark at the end of this text expressed exactly the central aspect according to which the strict criterion for mathematical object-languages had been liberalized: it is no longer required that it be general.

In keeping with this liberalization move, Carnap also gave up the requirement of generality of the significance criterion for descriptive languages. Accordingly, and this is the essential point, the use of a descriptive expression needs no more in all classes be justified as correct by an algorithm. Questions that have earlier been formulated as decision problems now become questions of choosing the form of language, i.e. questions 'of the establishment of rules of syntax and the investigation of the consequences of these' (*ibid.*, p. 164).

This is, then, some of the available evidence for the main thesis of this section: from both conceptual and historical point of view the first Carnapian liberalizing departure from the strict significance criterion is intimately related with general developments in the theory of formal systems, in particular, with Gödel's limitation theorems. On the other hand, there does not seem to exist any evidence that the liberalization move was mainly motivated by specific difficulties in the application of the strict criterion. In particular, there is no evidence that it had to do with the problems in the logic of disposition terms (Hempel [13], p. 3). Even after completing the liberalization by introducing the principle of tolerance in constructing languages, Carnap continued to treat the disposition terms like 'breakable' as on the same logical footing as terms for kinds or things ('horse'), for kind of substances ('iron'), and even for directly perceptible qualities ('warm', 'soft', 'sweet'). All these kinds of terms were considered to express properties of domains of things. No peculiarity of the disposition terms was mentioned (Carnap [8], p. 150).

Among the rules of syntax of liberalized descriptive languages, physical rules or P-rules are also admitted. Carnap distinguished several kinds of such rules. Some of these rules are theoretical postulates, such as the Maxwell's equations. The other kind is constituted by reduction sentences. Examples of these will be given below. Still an-

other kind contains the correspondence rules, such as the rule that says that the temperature of a gas is the mean kinetic energy of its molecules. In accordance with his general non-constructive position after 1933, Carnap considers the acceptance of P-rules not as a logico-philosophic problem but, to a large extent, a matter of convention and, hence, a question of expedience (Carnap [8], p. 180). All P-rules are used as non-logical rules for derivations and as means of introducing new terms. But the reduction sentences have an additional special usage: they serve as *partial* empirical decision procedures, based on empirical laws, for the concepts they introduce. The first treatment of reduction sentences is given in Carnap [7]. We shall now examine briefly how they work.

Let L_D be a first-order descriptive language, with unrestricted quantifiers, whose variables take their values in the domain of space-time points. Let Q_j, $j \neq 3$, be a sequence of descriptive predicate constants significant in L_D, and 'Q_3' a new constant whose intended interpretation is a disposition property. It can readily be shown that we cannot introduce 'Q_3' by explicit definitions. We can nevertheless introcude it into our L_D by admitting the following P-rules, called reduction sentences, as valid:

R_1 $\quad Q_1 \rightarrow (Q_2 \rightarrow Q_3)$

R_2 $\quad Q_4 \rightarrow (Q_5 \rightarrow \neg Q_3)$

In these formulas, 'Q_1' and 'Q_4' describe empirical conditions (and, in particular, test conditions) realized at a space-time point, 'Q_2' and 'Q_5' specify observable events (and, in particular, positive test results) at the same point, and 'Q_3' is the new constant. It is supposed that as a matter of empirical fact Q_1 is regularly followed by Q_2, and Q_4 by Q_5, equally without exception. It is clear that R_1 and R_2 constitute an *empirical* decision procedure for the problem of applicability of 'Q_3' (under the condition, of course, that we know the empirical procedures for the application of 'Q_1', 'Q_2', 'Q_4' and 'Q_5'). But this procedure is *not general*. For the class of points that satisfy the condition

(1) $\quad \sim ((Q_1 \wedge Q_2) \wedge (Q_4 \vee Q_5))$

the applicability of 'Q_3' is not decidable by means of R_1 and R_2. For these regions Q_3 is not determined, and hence, 'without meaning'.

(Carnap [7], p. 60). Now if the condition (1) were P-valid, 'Q_3' would be without any meaning at all in L_D.

Carnap also tried to formulate procedures for diminishing the indeterminacy of the application of newly introduced predicates by admitting as valid in L_D whole sequences $R^i_{1,2}$ of pairs of reduction sentences of the form

$$R^i_1 \qquad Q^i_1 \rightarrow (Q^i_2 \rightarrow Q_3)$$

$$R^i_2 \qquad Q^i_4 \rightarrow (Q^i_5 \rightarrow \neg Q_3)$$

where the sentences R^i_1 specify classes of space-time points $Q^i_{1,2}$ to which 'Q_3' applies and the sentences R^i_2 the classes of space-time points $Q^i_{4,5}$ to which it does not apply. Thus, a sequence $R^i_{1,2}$ of pairs of reduction sentences appears to constitute a decision procedure for the applicability of 'Q_3' stronger than the one based on any of its elements. In 1956, however, Carnap recognized that the use of a sequence $R^i_{1,2}$ for introducing a new property constant into L_D would violate the requirement of extensionality for L_D. As a matter of fact, the new constant introduced would remain intensional until an effective rule is laid down for the construction of the $R^i_{1,2}$. But, on the other hand, it is impossible to give such a rule, for to give it would be equivalent to specify the content and the order of discovery of all unknown empirical laws. Having recognized that a $R^i_{1,2}$ cannot possibly determine the exact extension of 'Q_3' and continuing to adopt a strictly extensional view on the meaning of descriptive constants, Carnap proposed that only single pairs of reduction sentences be used as (partial) definitions and that, in order to avoid ambiguity, we would call concepts so introduced by differents names, as has already beeen required by Bridgeman (Carnap [10], p. 64). This correction of an error in Carnap [7], shows in a particular clear way what the reduction pairs really are and what they are intended to be: decision procedures for the extensions of the constants they introduce.

In conclusion we can say the following: First, the strict criterion of cognitive significance was proposed by Carnap as a solution of decision problems for questions concerning the use of differents kinds of de-

scriptive expressions. In offering his solution, Carnap imitated solutions of decision problems for questions in mathematical language. By its very nature, the strict criterion belongs to the history of problem solving by effective methods. Second, the liberalization of the strict criterion was in the first place a consequence of internal limitations of effective methods discovered in the meantime. Third, the problems and methods peculiar to the traditional empiricism had little to do with the origins, the nature and the development of the Carnapian significance criterion.

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